On Intuitionistic Fuzzy Ideals In Boolean Like Semi Rings

P.Vijayalakshmi\textsuperscript{1}, R.Rajeswarir\textsuperscript{2}, P. Meenakshi\textsuperscript{3}
\textsuperscript{1} M.Sc Student, \textsuperscript{2,3} Assistant professor of Mathematics
\textsuperscript{1,2,3} A.P.C Mahalaxmi College for women, Thoothukudi, India.

Abstract- In this paper ,we introduce the concept of Intuitionistic fuzzy ideals in Boolean like semi rings and investigate some of their properties.

Keywords: Boolean like semi ring , Intuitionistic fuzzy set , Intuitionistic fuzzy ideal.

INTRODUCTION

Boolean like semi rings were introduced in role by K.Venkatesawaralu, B.V.N.Murthy and N.Amaranth during 2011. Boolean like rings of A.L.Foster arise naturally from general ring duality consideration and preserves many of the formal properties of Boolean ring . A Boolean like ring is commutative ring with unity and is of characteristics 2. It is clear that every Boolean ring is Boolean like ring but not conversely. The concept of a fuzzy subset of nonempty set was introduced by Zadeh. Fuzzy ideals of rings were introduced by Ziu , and it has been studied by several authors. The notion of fuzzy ideals and its properties were applied in various areas: Semi groups, Bck-algebras and semi rings. Y.B.Jun introduced the notion of fuzzy ideals in near rings.

In this paper we introduced of the intuitionistic fuzzy ideals in Boolean like semi ring and study some of its properties

2. PRELIMINARIES

Definition 2.1
A non-empty set R with two binary operation ‘+’ and ‘·’ is called a near-ring if
i) (R , +) is a group, (not necessarily abelian )
ii) (R , ·) is a semigroup.
iii) \( x \cdot (y + z) = x \cdot y + x \cdot z \) for all \( x,y,z \in R \).

Definition 2.2
A system \((R , +)\) is a Boolean semi ring iff the following property are hold
i) \((R , +)\) is an additive (abelian ) group (whose ‘zero’ will be denoted by ‘0’)
ii) \((R , ·)\) is a semigroup of idempotents in sense \( aa = a \) for all \( a \in R \).
iii) \( a \cdot (b + c) = ab + bc \) and
iv) \( abc = bac \) and \( a,b,c \in R \).

Definition 2.3
A non-empty set R together with two-binary operation + and · satisfying the following condition is called Boolean like semi ring
i) \((R , +)\) is an abelian group .
Definition 2.8
Let $X$ be a non-empty set and let $A = (\mu_A, v_A)$ and $B = (\mu_B, v_B)$ be IFSs in $X$. Then
1) $A \subseteq B$ if $\mu_A \leq \mu_B$ and $v_A \geq v_B$.
2) $A = B$ if $A \subseteq B$ and $B \subseteq A$.
3) $A^c = (\mu_A, v_A)$.
4) $A \cap B = (\mu_A \land \mu_B, v_A \lor v_B)$.
5) $A \cup B = (\mu_A \lor \mu_B, v_A \land v_B)$.
6) $A = (\mu_A, 1 - \mu_A)$, $A = (1 - \nu_B, v_B)$.

Definition 2.9
If $R$ and $R'$ are Boolean like semi rings a mapping $f: R \rightarrow R'$ is called be Boolean semi ring homomorphism (or simply homomorphism) of $R$ into $R'$ if
$f(a + b) = f(a) + f(b)$ and $f(ab) = f(a)f(b)$ for all $a, b \in R$.

Definition 2.10
A function $f: R \rightarrow R'$ is said to be Boolean like semi ring endomorphism if $f'$ is Boolean like semi ring homomorphism.

Definition 2.11
Let $f: X \rightarrow Y$ be mapping of Boolean like semi ring and $A$ be an intuitionistic fuzzy set of $Y$ then the map $f^{-1}(A)$ is the pre-image of $A$ under $f$ if
$\mu_{f^{-1}(A)}(x) = \mu_A(f(x))$ and
$\nu_{f^{-1}(A)}(x) = \nu_A(f(x))$ for all $x \in X$.

3. INTUITIONISTIC FUZZY IDEAL

Definition 3.1
An IFS $A = (\mu_A, v_A)$ in $R$ is called an intuitionistic fuzzy left (resp. right) ideal of Boolean like semi ring $R$ if
1) $\mu_A(x \land y) \geq (\mu_A(x) \land \mu_A(y))$.
2) $\mu_A(ra) \geq \mu_A(a)$.
3) $\mu_A((r + a)s + rs) \geq \mu_A(a)$.
4) $\nu_A(x \lor y) \leq (\nu_A(x) \lor \nu_A(y))$.
5) $\nu_A(ra) \leq \nu_A(a)$.
6) $\nu_A((r + a)s + rs) \leq \nu_A(a)$.
$\forall \ r, r, a, s, x, y \in R$.

Example 3.2
\[
\begin{array}{cccc}
+ & 0 & a & b & c \\
0 & 0 & a & b & c \\
a & a & 0 & c & b \\
b & c & 0 & a & b \\
c & a & b & a & 0 \\
\end{array}
\]

Clearly it is a Boolean – like – semi ring. Let $A = (\mu_A, v_A)$ be an intuitionistic fuzzy ideal defined on Boolean like semi ring $R$ by $\mu_A(0) = 0.7, \mu_A(a) = 0.6, \mu_A(b) = 0.5, \mu_A(c) = 0.5$ and $v_A(x) = 0.6$ for every $x \in R$. Then $A = (\mu_A, v_A)$ is an intuitionistic fuzzy ideal of $R$.

Theorem 3.3
If $A$ is an ideal of a Boolean like semiring $R$, then the IFS $\hat{A} = (\hat{\chi_A}, \hat{\mu_A})$ is an intuitionistic fuzzy ideal of $R$.

Proof
Let $x, y, r, s \in R$.
If $x, y, u, v, a \in A$ then $x - y \in A$, $ra \in A$, and $r(a)s + rs \in A$. Since $A$ is an ideal of $R$.

Hence $\chi_A(x - y) = 1 \geq (\chi_A(x) \land \chi_A(y))$.
$\chi_A(ra) = 1 \geq \chi_A(a)$.
$\chi_A((r + a)s + rs) = 1 \geq \chi_A(a)$.

Also we have,
$0 = 1 - \chi_A(x - y) = \overline{x_A(x - y)} \\
\leq (\overline{x_A(x)} \lor \overline{x_A(y)}) \\
0 = 1 - \chi_A(ra) = \overline{x_A(ra)} \\
= \overline{\chi_A((r + a)s + rs)} \\
= \overline{\chi_A(a)}$.

If $a \in A$ and $y \in A$ then $\chi_A(x) = 0$ (or) $\chi_A(y) = 0$.

Thus we have,
$\chi_A(x - y) \geq (\chi_A(x) \land \chi_A(y))$.
$\chi_A(ra) \geq \chi_A(a)$.
$\chi_A((r + a)s + rs) \geq \chi_A(a)$.

Also,
$\overline{x_A(x - y)} = 1 - \chi_A(x - y) \\
\overline{x_A((r + a)s + rs)} = 1 - \chi_A((r + a)s + rs)$.

Example 3.2

This completes the proof.

Definition 3.4
An intuitionistic fuzzy left (resp. right) ideal in Boolean like semi ring $R$ is said to be normal if $\mu_A(0) = 1$ and $\nu_A(0) = 0$.

Theorem 3.5
Let $A = (\mu_A, v_A)$ be an intuitionistic fuzzy left (resp. right) ideal of Boolean like semi ring $R$ and let $\mu_A^x(a) = \mu_A(a) + 1 - \mu_A(0)$ and $v_A^x(a) = v_A(a) - v_A(0)$ if...
\( \mu_A(x) + \nu_A(x) \leq 1 \) for all \( x \in M \). Then \( A^+ = (\mu_A^+, \nu_A^+) \) is an intuitionistic fuzzy left (resp. right) ideal of \( R \).

**Proof:**

We first observe that \( \mu_A^+(0) = 1 \) and \( \nu_A^+(0) = 0 \) and

\[ \mu_A^+(0), \nu_A^+(0) \in [0,1] \text{ for all } x \in R. \]

Hence, \( A^+ = (\mu_A^+, \nu_A^+) \) is a normal intuitionistic fuzzy set.

To prove: it is an intuitionistic fuzzy left (resp. right) ideal.

Let \( x, y, r, a, s \in R, \) Then

\[ \mu_A^+(x-y) = \mu_A(x-y) + 1 - \mu_A(0) \]

\[ \geq \{ \mu_A(x) \land \mu_A(y) \} + 1 - \mu_A(0) \]

\[ \geq \{ \mu_A(x) + 1 - \mu_A(0) \} \land \{ \mu_A(y) + 1 - \mu_A(0) \} \]

\[ \geq \mu_A^+(x) \land \mu_A^+(y) \]

\[ \mu_A^+(ra) = \mu_A(ra) + 1 - \mu_A(0) \]

\[ \geq \mu_A(a) + 1 - \mu_A(0) \]

\[ \mu_A^+((r + a)s + rs) \]

\[ = \mu_A((r + a)s + rs) + 1 - \mu_A(0) \]

\[ \geq \mu_A(a) + 1 - \mu_A(0) \]

\[ \nu_A^+(x-y) = \nu_A(x-y) - \nu_A(0) \]

\[ \leq \{ \nu_A(x) \lor \nu_A(y) \} - \nu_A(0) = \{ \nu_A(x) - \nu_A(0) \} \lor \{ \nu_A(y) - \nu_A(0) \} \]

\[ \nu_A^+(ra) = \nu_A(ra) - \nu_A(0) \leq \nu_A^+(a) \]

\[ \nu_A^+((r + a)s + rs) \]

\[ = \nu_A((r + a)s + rs) - \nu_A(0) \]

\[ \leq \nu_A(a) - \nu_A(0) \]

\[ = \nu_A^+(a) \]

This shows that \( A^+ \) is an intuitionistic fuzzy left ideal of \( R \).

So \( A^+ \) is a normal intuitionistic fuzzy left ideal of \( R \).

**Theorem 3.6**

If the IFS \( A = (\mu_A, \nu_A) \) is an intuitionistic fuzzy left (resp. right) ideal of \( R \), then the set \( R_A = \{ x \in R | \mu_A(x) = \mu_A(0) \text{ and } \nu_A(x) = \nu_A(0) \} \) is an ideal of \( R \).

**Proof:**

Let \( y, a \in R_A \), then

\[ \mu_A(x) = \mu_A(y) = \mu_A(0) \text{ and } \nu_A(x) = \nu_A(y) = \nu_A(0) \]

Since \( A \) is an intuitionistic fuzzy ideal of \( R \) it follows.

\[ \mu_A(x-y) \geq \{ \mu_A(x) \land \mu_A(y) \} \]

\[ = \{ \mu_A(0) \land \mu_A(0) \} \]

Hence \( \mu_A(x-y) = \mu_A(0) \)

\[ \mu_A(ra) \geq \mu_A(a) \]

\[ = \mu_A(0) \]

Let \( r, s \in R \)

Therefore, \( \mu_A((r + a)s + rs) \geq \mu_A(a) \)

\[ = \mu_A(0) \]

Similarly,

\[ \nu_A(x-y) \leq \{ \nu_A(x) \lor \nu_A(y) \} \]

\[ = \{ \nu_A(0) \lor \nu_A(0) \} \]

\[ = \nu_A(0) \]

Hence \( \nu_A(x-y) = \nu_A(0) \)

So \( x - y \in R_A \)

\[ v_A((r + a)s + rs) \leq v_A(a) \]

Hence \( v_A(ra) = v_A(0) \) and \( v_A(ra) = v_A(0) \)

So \( x \in R_A \)

**Theorem 3.7**

Let \( A \) be an intuitionistic fuzzy left (resp. right) ideal of a Boolean like semi ring \( R \). For each pair \( (t, s) \in [0,1] \), the level set \( A_{(t,s)} \) is an ideal of \( R \).

**Proof:**

Let \( x, y, a \in A_{(t,s)} \), then

\[ \mu_A(x) \geq t, \ \mu_A(y) \geq t \land \mu_A(a) \geq t \]

\[ v_A(x) \leq s, v_A(y) \leq s \text{ and } v_A(a) \leq s. \]

Since \( A \) is an intuitionistic fuzzy left (resp. right) ideal we have,

\[ \mu_A(x-y) \geq \{ \mu_A(x) \land \mu_A(y) \} \geq t \]

\[ \mu_A(ra) \geq \mu_A(a) \geq t \]

Then \( \mu_A((r + a)s + rs) \geq \mu_A(a) \geq t \)

\[ v_A(x-y) \leq \{ v_A(x) \lor v_A(y) \} \leq s \]

So \( x-y \in A_{(t,s)} \)

\[ v_A(ra) \leq v_A(a) \leq s \]

So \( ra \in A_{(t,s)} \)

\[ v_A((r + a)s + rs) \leq v_A(a) \leq s \]

So \( (r + a)s + rs \in A_{(t,s)} \). Hence \( A_{(t,s)} \) is an ideal of \( R \).

**Theorem 3.8**

Let \( f \) be a Boolean semi-ring homomorphism of \( R \) if the IFS \( A = (\mu_A, \nu_A) \) is an intuitionistic fuzzy ideal of \( R \), then \( B = (\mu_{f^{-1}}(A), \nu_{f^{-1}}(A)) \) is an intuitionistic fuzzy left (resp. right) ideal of \( R \).

**Proof:**

For any \( x, y, r, a, s \in R \), we have

\[ \mu_{f^{-1}}(A)(x-y) = \mu_A(f(x-y)) \]

\[ = \mu_A(f(x) - f(y)) \]

\[ \geq \{ \mu_A(f(x)) \land \mu_A(f(y)) \} \]

\[ = \{ \mu_{f^{-1}}(A)(x) \land \mu_{f^{-1}}(A)(y) \} \]

\[ = \mu_{f^{-1}}(A)(ra) = \mu_A(f(ra)) \]

\[ = \mu_A(f(r)f(a)) \geq \mu_A(f(a)) \]

\[ = \mu_{f^{-1}}(A)(a) \]

\[ \mu_{f^{-1}}(A)((r + a)s + rs) \]

\[ = \mu_A(f(r + a)s + rs) \]

\[ = \mu_A(f(r + a) + f(rs)) \]

\[ \geq \mu_A(f(a)) \]

\[ = \mu_{f^{-1}}(A)(a) \]

Similarly,

\[ v_{f^{-1}}(A)(x-y) = v_A(f(x-y)) \]

\[ = v_A(f(x) - f(y)) \]

\[ = v_{f^{-1}}(A)(x) \lor v_{f^{-1}}(A)(y) \]
\[ v_{f^{-1}(A)}(x) \vee v_{f^{-1}(A)}(a) = v_{A}(f(ra)) \]
\[ \leq v_{A}(f(a)) \]
\[ = v_{f^{-1}(A)}(a) \]

\[ v_{f^{-1}(A)}(r + a)s + rs) = v_{A}(f(r + a)s + rs) \]
\[ \leq v_{A}(f(a)) \]
\[ = v_{f^{-1}(A)}(a) \]

Hence B is an intuitionistic fuzzy left(resp. right) ideal of R.

**Theorem 3.9**

Intersection of a non-empty collection of intuitionistic fuzzy left (resp. right) ideals of a Boolean like semi ring R is an intuitionistic fuzzy left(resp. right) ideal of R.

**Proof:**

Let \( R \) be a Boolean like semi ring.

Let \( A = \{\mu_{A_i}, v_{A_i}/i \in I\} \) be the family of intuitionistic ideal of Rand let \( x, y, r, a, s \in R \).

Then we have,

i) \( (\cap_{i \in I} \mu_{A_i})(x - y) = \inf_{i \in I} \{\mu_{A_i}(x - y)\} \)
\[ \geq \inf_{i \in I} \{\mu_{A_i}(x) \wedge \mu_{A_i}(y)\} \]
\[ = \bigcap_{i \in I} \mu_{A_i}(x) \wedge \bigcap_{i \in I} \mu_{A_i}(y) \]

ii) \( (\cap_{i \in I} \mu_{A_i})(ra) = \inf_{i \in I} \{\mu_{A_i}(ra)\} \)
\[ \geq \inf_{i \in I} \{\mu_{A_i}(a)\} \]
\[ = (\cap_{i \in I} \mu_{A_i})(a) \]

iii) \( (\cap_{i \in I} \mu_{A_i})(r + a)s + rs) \)
\[ \geq \inf_{i \in I} \{\mu_{A_i}(r + a)s + rs)\} \]
\[ = (\cap_{i \in I} \mu_{A_i})(a) \]

Similarly,

i) \( (\cap_{i \in I} v_{A_i})(x - y) = \inf_{i \in I} \{v_{A_i}(x - y)\} \)
\[ \leq \inf_{i \in I} \{v_{A_i}(x) \vee v_{A_i}(x)\} \]
\[ = \bigcap_{i \in I} v_{A_i}(x) \vee \bigcap_{i \in I} v_{A_i}(y) \]

ii) \( (\cap_{i \in I} v_{A_i})(ra) = \inf_{i \in I} \{v_{A_i}(ra)\} \)
\[ \leq \inf_{i \in I} \{v_{A_i}(a)\} \]
\[ = (\cap_{i \in I} v_{A_i})(a) \]

iii) \( (\cap_{i \in I} v_{A_i})(r + a)s + rs) \)
\[ = \inf_{i \in I} \{v_{A_i}(r + a)s + rs)\} \]
\[ \leq \inf_{i \in I} \{v_{A_i}(a)\} \]
\[ = (\cap_{i \in I} v_{A_i})(a) \]

**REFERENCES**