Some Graphs On Near Divisor Cordial-II

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Abstract: - A Near divisor cordial labeling of a graph G with vertex set V is a bijection f from V to {1,2,...,│V│-1,│V│+1},such that if each edge uv is assigned the label 1 if f (u) divides f (v) (or) if f (v) divides f (u) and 0 otherwise, then the number of edges labelled with 0 and the number of edges labelled with 1 differ by almost 1. If a graph admits Near divisor cordial labeling then it is called Near divisor cordial graph. In this paper , We proved graphs such as J(n+1,n),Sn,Bn,m2,K1,n,S(K1,n),K2,n,K3,n, <K1,n(1),K1,n(2)>and<K1,n(1),K1,n(2),K1,n(3)> are Near divisor cordial (NDC). AMS Mathematics subject classification 2010: 05c78.

Keywords
Cordial labelling , Divisor cordial labelling and Near divisor cordial labelling.

1. INTRODUCTION

By a graph , we mean a finite undirected graphs without loops and multiple edges for terms not defined here.We refer to Harary [3]

Definition 1.1 [1]:
Let G = (V,E) be a graph. A mapping f : V(G) → { 0,1 } is called binary vertex labelling of G and f (v) is called the label of the vertex v of G under f.
Cahit [1] defined cordial labelling as follows
Definition 1.2:
A binary vertex labelling of a graph G is called a cordial labelling if │v f (0) – v f (1) │≤ 1 and │e f (0) – e f (1) │≤ 1 . A graph G is cordial if it admits cordial labeling.
Here f * : E(G) → { 0 , 1 } is given by f *(e) = │f (u) – f (v) │. v f (0) , v f (1) be the number of vertices of G having labels 0 and 1 respectively under f and e f (0) , e f (1) be the number of edges of G having 0 and 1 respectively under f*.
The concept of divisor cordial labelling is introduced by R.Varatharajan, S.Navaneetha Krishnan and K.Nagarajan [5] and defined as follows:
Definition 1.3 [5] :
Let G = (V,E) be a simple graph and f : v → {1,2,.....,|V|} be a bijection. For each edge uv , assign the label 1 if either f (u) | f (v) or if f (v) | f(u) and the label 0 otherwise. f is called divisor cordial labelling if │e f (0) – e f (1) │≤ 1.
The concept of Near graceful labelling is introduced by Frucht [4] with edge labelling {1,2,...,q-1,q+1}.Motivated by the above definitions, we introduce the concept called Near divisor cordial.
2. Main Results:
Definition 2.1:
Let G = (V,E) be a simple graph and f : V (G) → {1,2,....,|V|-1,|V|+1} be a bijection. For each edge uv, assign the label 1 if either f (u) | f (v) or f (v) | f(u) and the label 0 otherwise. f is called Near divisor cordial labelling if │e f (0) – e f (1) │≤ 1.
Note that K7 is not divisor cordial but it is Near divisor cordial and K1,2m is divisor cordial but it is not Near divisor cordial. Hence the above definition is meaningful.
The following definitions are useful for proving theorems.
Definition 2.2 : For integers m,n ≥ 0 , we consider the graph jellyfish J(m,n) with vertex set V(J(m,n))={u,v,x,y} U { x1,x2,. . . . . . . ,xm } U {y1,y2, ... ,yn } and the edge set E(J(m,n)) = {(u,x) , (u,y), (u,v), (v,x), (v,y)} U {(xi,x) / 1≤ i ≤ m} U {(yi,y) / 1≤ i ≤ n}.
Definition 2.3: The graph Pn+ K1 is called a shell
Definition 2.4 : The Bistar Bm,n is the graph obtained from K2 by identifying the center vertices of K1,m and k1,n at the vertices of K2 respectively. Bm,m is often denoted by B(m).
The Complete bipartite graph K1,n is called a Star Graph and it is demoted by Sm.
S(K1,n) the sub division of the star k 1,n is a tree obtained from the star k 1,n by adding a new pendent edge to each of the existing n pendent vertices.
Definition 2.5: Consider two stars K1,n(1) and K1,n(2) .Then G = < K1,n(1) ,K1,n(2) > is the graph obtained by joining apex vertices of star to a new vertex x.
Note that G has 2n+3 vertices and 2n+2 edges.
Definition 2.6: Consider t copies of stars namely K1,n(1) ,K1,n(2) ,. . . . . , K1,n(t) . Then G = < K1,n(1) ,K1,n(2) , ... , K1,n(t) > is the graph obtained by joining apex vertices of each K1,n(m-1) and K1,n(m) to a new vertex xm-1 where 2 ≤ m ≤ t.
Note that G has t(n+2)-1 vertices and t(n+2)-2 edges.
THEOREM 2.7:
The graph J(n+1,n) is Near divisor cordial
Proof:
Let V(J(n+1,n))={u1,u2,...................,un,w1,w2, w3, w4, v1,v2,v3,.............,vn+1}

E(J(n+1,n)) = { w1w2, w2w3, w3w4, w4w1,w2w4}U{ w3ui / $1 \le i \le n$ }U{w1vi / $1 \le i \le n+1$}
Define f (w1) = 1 and f (w3) = S such that s is the largest prime number such that $S \le 2n+6$ and $S \neq 2n+5$
Label the remaining vertices from {2,3,............,s-1,s+1,........,2n+4,2n+6} in that order.
Then ,e f (0) = e f (1) = k where $k = \frac{2n+6}{2}$
Hence $|e f(0) - e f(1)| = 0$
Hence, the graph J(n+1,n) is a Near divisor cordial

Example 2.8:

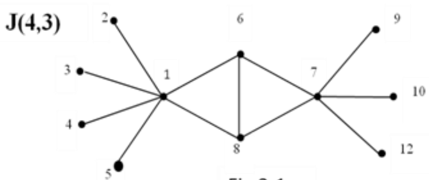


Fig 2.1

THEOREM 2.9:
The shell Sn is a Near divisor cordial
Proof:
Let V(Sn) = {v0, v1, v2, ... vn-1}
E(Sn)={v0vi , $1 \le i \le n-1$ } and i≠n} U { vivi+1 , $1 \le i \le n-2$ }
Fix f (v0) =1
Label the remaining vertices from {2,3,4,........,n-1,n+1} in that order.
Then e f (0)=n-2 , e f (1)=n-1
Hence $|e f(0) - e f(1)| = 1$
Hence, Sn is a Near divisor cordial

Example 2.10:

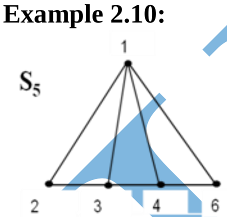


Fig 2.2

THEOREM 2.11:
The Graph Bn,m2 is Near divisor cordial
Proof: |V(Bn,m2)|=n+m+2and |E(Bn,m2)|=4n+1
Always fix f(v0)=1 and f(u0)=S, where s is the largest prime such that S≤n+m+3 and S≠n+m+2.
Then label the remaining vertices from {2,3,.....,n+m+1,n+m+3}
Then e f (0) = n + m , e f (1) = n+m+1
Hence $|e f(0) - e f(1)| = 1$
Hence, The Graph Bn, m2 is Near divisor cordial
Example 2.12:

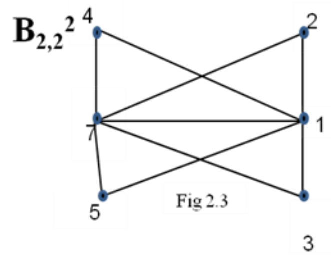


Fig 2.3

THEOREM 2.13:
The star graph K1,n is Near divisor cordial iff n is odd, n ≥ 5
Proof:
Let V (K1,n) = {v1,v2,v3,. . . . . . ,vn } and E (K1,n) = { vvi : $1 \le i \le n$} and n=4.
Now assign label 2 to the vertex v and label the remaining vertices v1, v2, v3, ... ,vn by 1,3,4,. . . . . , n-1 and n+1 respectively.
We have, e f (0) = k+1 , e f (1) = k , where n= 2k+1
Hence , Hence $|e f(0) - e f(1)| = 1$.
Therefore, K1,n is Near divisor cordial for n is odd and $n \ge 5$, n = 4,6
Conversely,
Suppose K1,n is Near divisor cordial
Suppose n is even and $n \ge 8$
Let n = 2k , there are k+1 even numbers and k odd numbers as labels.
Assigning any odd number to the central vertex as label, then it does not satisfy the condition
$|e f(0) - e f(1)| \le 1$.
If f (v) = 2 for a labelling f in that case also e f (1) = k+1 and e f (0) = k-1. It can be easily verifies that by assigning any even number > 2 to the central vertex as label, then it does not satisfy the condition $|e f(0) - e f(1)| \le 1$.
∴ K1,n is not Near divisor cordial When n is even & $n \ge 8$.
Clearly, K1,n ≅ P3 is not near divisor cordial.
Therefore if K1,n is Near divisor cordial then n should be odd and $n \ge 3$ and n=4,6.

Example 2.14:

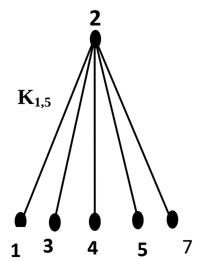


fig 2.4

THEOREM 2.15:
S (K1, n) the sub division of the star k 1,n is near divisor cordial
Proof:
Let V (S(K1,n)) = { v , vi , ui : $1 \le i \le n$} and
E (S(K1,n)) = {vvi ,viui : $1 \le i \le n$ }
Define f by f (v) = 1
f (vi) = 2i ($1 \le i \le n$)
f (ui) = 2i+1 ($1 \le i \le n-1$)
and f (un) = 2i+2 , where i = n
here e f (0) = e f (1) = n
Hence │e f (0) – e f (1)│= 0
Therefore, S(K1,n) is near divisor cordial.
Example 2.16:

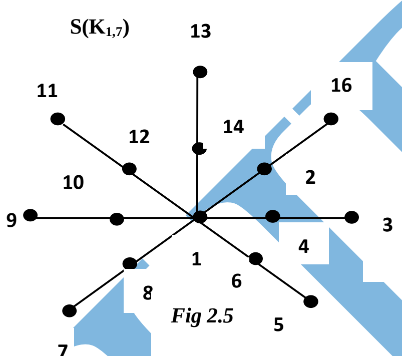


Fig 2.5

THEOREM 2.17:
The complete bipartite graph K2,n is near divisor cordial.
proof:
Let V (K2,n) = V1UV2
Such that│V(K2,n)│=n+2 and │E(K2,n)│ = 2n.
V1={x1,x2} and
V2= {v1,v2,v3,..............,vn}.Now assign the label 1 to x1 and the largest prime number S to x2 such that $S \le n+2$, $S \ne n+1$.Then the remaining vertices y1,y2,y3,..............,yn is labelled from {2,3,4,..........,n+1,n+3}-{s}.
Clearly, e f (0) = e f (1) = n
Hence │e f (0) – e f (1)│=0
Hence, K 2,n is a Near divisor cordial

Example 2.18:

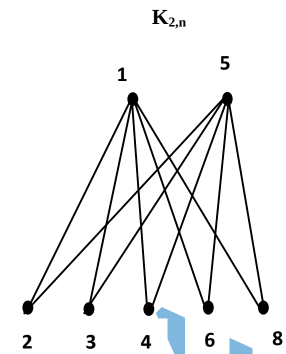
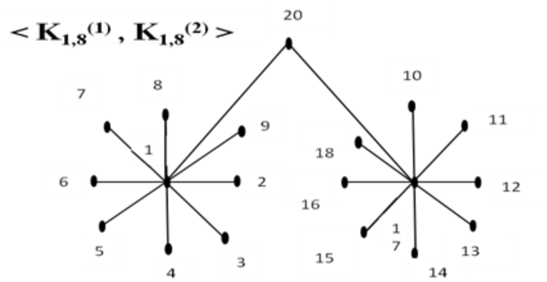


fig 2.6

THEOREM 2.19:
The complete bipartite graph K3,n is Near divisor cordial, n is odd.
proof:
Let V (K3,n) = V1UV2
Such that │V(K3,n)│ = n+3 and │E(K3,n)=3n.V1 ={ x1, x2, x3} and V2 ={v1,v2,v3, ...,vn }.Now define f (x1) = 1 , f (x2) = 2 and f (x3) = s ,where s is the largest prime number such that $s \le n+4$ and $s \ne n+3$.
And assign the remaining labels to the vertices y1, y2, y3, ... , yn
Then, e f (0) = n ,e f (1) = n-1
Hence │e f (0) – e f (1)│= 1
Thus, K3,n is a Near divisor cordial.
Remark 2.21:
For Km,n , $m \ge 4$, e f (0) value increases drastically than e f (1) and it is true for any Near divisor cordial labelling f except for some particular values of m & n.

THEOREM 2.22:
The graph G = <K1,n(1),K1,n(2) > is Near divisor cordial
Proof :
Let v1(1) ,v2(1) , ... ,vn(1) be the pendent vertices of K1,n(1) and v1(2) ,v2(2) , ... , vn(2) be the pendent vertices of K1,n(2)
Let c1 and c2 be the apex vertices of K1,n(1) and K1,n(2) respectively and they are adjacent to a common vertex w.│V (G)│ = 2n+3 │E (G)│ = 2n+2. Let f : V (G) → { 1,2,3, ... ,2n+2 , 2n+4 }
Now, assign the label 1 to c1 and the largest prime number S such that $S \le 2n+4$ (and $S \ne 2n+3$) to c2 and the remaining numbers to be labelled to the remaining vertices of G. Since 1 divides any integer, and S does not divide any integer, then e f (0) = n+1 and e f (1) = n+1
Hence, │e f (0) – e f (1)│=0.
Hence, the graph G = < K1,n(1) ,K1,n(2) > is Near divisor cSordial

Example 2.23:



Theorem 2.24:

The graph G = < K1,n(1) ,K1,n(2) , K1,n(3) > is Near divisor cordial

Proof :

Let v1(1) ,v2(1) , ... , vn(1) be the pendent vertices of K1,n(1) and v1(2),v2(2), ... ,vn(2) be the pendent vertices of K1,n(2) and v1(3),v2(3), ... ,vn(3) be the pendent vertices of K1,n(3).

Let c1 and c2 and c3 be the apex vertices of K1,n(1) and K1,n(2) and K1,n(3) respectively and they are adjacent to a common vertex w1 and w2.such that w1 is adjacent to c1 and c2 and w2 is adjacent to c2 and c3.

Note that G has 3n+5 vertices and 3n+4 edges.

Case 1: n is odd

Now assign the label 1 to c1, 2 to c2 and S to c3 where S is the largest prime number such that S ≤ 3n+5 (and S ≠ 3n+4). Then assign the remaining numbers to the pendent vertices, we get,

ef(1) = $\frac{3n+5}{2}$ and ef(0) = $\frac{3n+3}{2}$ (See fig)

Then, $|$ e f (0) – e f (1) $|$ = 1.

Case 2: n is even

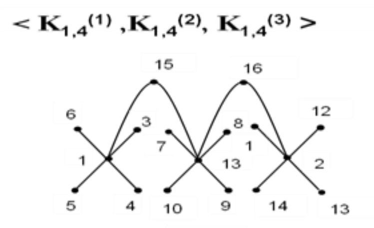
Now assign the label 1 to c1 , S to c2 , where S is the largest prime number such that S≤ 3n+5 and 2 to c3 . Then assign the remaining numbers to the pendent vertices in such a way that $\frac{n+1}{2}$ vertices adjacent to C3 is assigned even numbers and remaining $\frac{n+1}{2}$ vertices adjacent to C3 is assigned odd numbers and the remaining labels are assigned to the left over pendent vertices.

We get,e f (1) = $\left\lfloor\frac{3n+4}{2}\right\rfloor$ = e f (0) (See fig)

Then, $|$ e f (0) – e f (1) $|$ =1.

Hence, The graph G = < K1,n(1) , K1,n(2) , K1,n(3) > is Near divisor cordial.

Example 2.25:



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