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A Study on Balking and Re-Service in A Bulk Queue with Two Types of Heterogeneous Service in an Out Patient Department

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Abstract- **We analyze a bulk queue with a single server with two types of heterogeneous service. At the beginning of the service, a customer has the option to choose either type 1 or type 2 service. We added the concept of balking and re-service in this study. Once a service is completed the customer may leave the system or he has the option to demand re-service. In this study we have derived the steady state queue size distribution and some particular cases have been developed.**

Key words: bulk arrival, heterogeneous service, balking, re-seervice, ,queue size

INTRODUCTION

In a healthcare settings, when accidents or emergency situations such as terrorist events, environmental disasters, or mass casualty events involve multiple people, it is often that the parties involved arrive to the hospital in bulk. Moreover, these people can also be rejected and rerouted to neighboring hospitals in batches if they exceed the hospital's capacity. In our model we assume patients arriving in bulk and the system is providing in parallel two types of general heterogeneous service, the patients can choose any one type of service. We can find many real life applications of this model. There may be situations where re-service is desired, for instance while visiting a doctor, the patient may be recommended for some investigations, after which he may need to see the doctor again while in some situations offering public service a customer may find his service un satisfactory and consequently demand re-service. Here we have discussed the case of optional re-service. In this case a customer has the choice of selecting any one of the two types of heterogeneous service, subsequently has the option to repeat the service taken by him or depart from the system

Here we assume that patients arrive in bulk of variable size according to a compound poisson process.

Let $\lambda a_i dt$ (i = 1, 2 …) be the first order probability of arrival of 'i' patients in bulk in the system at a short interval of time (t, t + dt) where $O \le a_i \le 1$, $\sum_{i=1}^{\infty} a_i = 1$, $\lambda > 0$ is the mean bulk arrival rate.

We consider the case when there is a single doctor providing parallel service of two types on a first come first served basis (FCFS). At the start of the service, each patient has the choice of choosing either first service (first – Aid) with probability μ_1 or can choose second service (emergency care) with probability μ_2 and $\mu_1 + \mu_2 = 1$.

We assume that the random variable of service time w_j (j = 1, 2) of the j^{th} kind of service follows a general probability law with distribution function $V_j(w_j)$, $v(w_j)$ is the probability density function and $E(w_j^k)$ is the k^{th} moment (k = 1, 2 …) of service time, j = 1, 2

Let $\gamma_j(x)$ be the conditional probability of type j service during the period $(x, x+dx]$ given that the elapsed time is x, so that

$$\gamma_j(x) = \frac{v_j(x)}{1 - V_j(x)},\ j = 1, 2 \qquad \text{….. (1)}$$

$$v_j(x) = \gamma_j(w_j)\, e^{-\int_0^w \gamma_j(x)dx},\ j = 1, 2 \qquad \text{….. (2)}$$

Once the service of a patient is complete, the server (Doctor) leave the Hospital with probability q or may continue to serve the next patients with probability 1-q or may remain idle in the Hospital, even if there is no patient requiring service.

Also we assume that (1-d) $(0 \le d \le 1)$ is the probability that an bulk arriving balks during the period when the doctor is busy (available on the system). As soon as service (of any one kind) is complete he has the option to leave the Hospital or join the hospital for re-service, if necessary. We assume that probability of repeating type j service as r_j and leaving the Hospital without re-service as $(1-r_j)$, j = 1, 2. We consider that either service may be repeated only once.

Definitions and Notations:

Assuming that steady state exists, we define

i) $P_{n,j}(x)$ = Probability that there are n(≥ 1) patients in the Hospital including one patient in type j service, j = 1, 2 and elapsed service time is x.

ii) $P_{n,j}(x) = \int_0^\infty P_{n,j}(x)dx$ is the corresponding steady state probability irrespective of elapsed time x.

iii) $D_{n,j}(x)$ = Probability that there are n (≥ 1) patients in the Hospital including one patient who is repeating type j service j = 1, 2 and elapsed service time is x.

iv) $D_{n,j} = \int_0^\infty D_{n,j}(x)dx$ is the corresponding steady state probability irrespective of elapsed service time x.

v) E = steady state probability of the Doctor is idle but available in the Hospital and there is no patient requiring service.

The probability Generating Functions are

i) $P_j(x, z) = \sum_{n=1}^{\infty} z^n P_{n,j}(x)$

ii) $P_j(z) = \sum_{n=1}^{\infty} z^n P_{n,j}$; $\quad |z| \le 1;\ j = 1, 2$

iii) $D_j(x, z) = \sum_{n=1}^{\infty} z^n D_{n,j}(x)$

iv) $D_j(z) = \sum_{n=1}^{\infty} z^n D_{n,j}$; $\quad |z| \le 1;\ j = 1, 2$

v) $A(z) = \sum_{i=1}^{\infty} z^i a_i$; $\quad |z| \le 1$

Equations Governing the system

Let us define the steady state equations for our model as

$$\frac{d}{dx} P_{n,1}(x) + (\lambda + \gamma_1(x))\, P_{n,1}(x) = \lambda(1-d)P_{n,1}(x) + d\lambda \sum_{i=1}^{n} a_i P_{n-i,1}(x) \qquad \text{..... (3)}$$

$$\frac{d}{dx} P_{n,2}(x) + (\lambda + \gamma_2(x))\, P_{n,2}(x) = \lambda(1-d)P_{n,2}(x) + d\lambda \sum_{i=1}^{n} a_i P_{n-i,2}(x) \qquad \text{..... (4)}$$

$$\frac{d}{dx} D_{n,1}(x) + (\lambda + \gamma_1(x))\, D_{n,1}(x) = \lambda(1-d)D_{n,1}(x) + d\lambda \sum_{i=1}^{n} a_i D_{n-i,1}(x) \qquad \text{..... (5)}$$

$$\frac{d}{dx} D_{n,2}(x) + (\lambda + \gamma_2(x))\, D_{n,2}(x) = \lambda(1-d)D_{n,2}(x) + d\lambda \sum_{i=1}^{n} a_i D_{n-i,2}(x) \qquad \text{..... (6)}$$

$$\lambda E = \lambda(1-d)E + (1-q)\left[(1-r_1)\int_0^\infty P_{1,1}(x)\gamma_1(x)dx + (1-r_2)\int_0^\infty P_{1,2}(x)\gamma_2(x)dx\right] + (1-q)\left[\int_0^\infty D_{1,1}(x)\gamma_1(x)dx + \int_0^\infty D_{1,2}(x)\gamma_2(x)dx\right] \qquad \text{..... (7)}$$

where P0, j (x) = 0, D0, j (x) = 0, j =1, 2 for (3), (4), (5) and (6)
The boundary conditions for solving the above differential equations at x = 0 are

$$P_{n,1}(0) = (1-q)\mu_1\left[(1-r_1)\int_0^\infty P_{n+1,1}(x)\gamma_1(x)dx + (1-r_2)\int_0^\infty P_{n+1,2}(x)\gamma_2(x)dx\right] + (1-q)\mu_1\left[\int_0^\infty D_{n+1,1}(x)\gamma_1(x)dx + \int_0^\infty D_{n+1,2}(x)\gamma_2(x)dx\right] + \lambda d\mu_1 a_n E, \quad n \ge 1 \qquad \text{..... (8)}$$

$$P_{n,2}(0) = (1-q)\mu_2\left[(1-r_1)\int_0^\infty P_{n+1,1}(x)\gamma_1(x)dx + (1-r_2)\int_0^\infty P_{n+1,2}(x)\gamma_2(x)dx\right] +$$

$$(1-q)\mu_2\left[\int_0^\infty D_{n+1,1}(x)\gamma_1(x)dx + \int_0^\infty D_{n+1,2}(x)\gamma_2(x)dx\right] + \lambda d\mu_2 a_n E, \quad n \geq 1 \qquad \text{..... (9)}$$

$$D_{n,1}(0) = r_1\int_0^\infty P_{n,1}(x)\gamma_1(x)dx \qquad \text{..... (10)}$$

$$D_{n,2}(0) = r_2\int_0^\infty P_{n,2}(x)\gamma_2(x)dx \qquad \text{..... (11)}$$

and the normalizing condition

$$E + \sum_{j=1}^{2}\sum_{n=1}^{\infty}\int_0^\infty P_{n,j}(x)\,dx + \sum_{j=1}^{2}\sum_{n=1}^{\infty}\int_0^\infty D_{n,j}(x)\,dx = 1 \qquad \text{..... (12)}$$

Queue Size Distribution at Random Epoch

Now let us multiply Equations (3) – (6) by z^n, and taking summations over all possible values of n and simplifying we get

$$\frac{d}{dx}P_1(x,z) + \{d(\lambda - \lambda A(z)) + \gamma_1(x)\}\,P_1(x,z) = 0 \qquad \text{..... (13)}$$

$$\frac{d}{dx}P_2(x,z) + \{d(\lambda - \lambda A(z)) + \gamma_2(x)\}\,P_2(x,z) = 0 \qquad \text{..... (14)}$$

$$\frac{d}{dx}D_1(x,z) + \{d(\lambda - \lambda A(z)) + \gamma_1(x)\}\,D_1(x,z) = 0 \qquad \text{..... (15)}$$

$$\frac{d}{dx}D_2(x,z) + \{d(\lambda - \lambda A(z)) + \gamma_2(x)\}\,D_2(x,z) = 0 \qquad \text{..... (16)}$$

We now Integrate equations (13) – (16) between limits 0 and x and obtain

$$P_1(x,z) = P_1(0,z)\,e^{-d\lambda(1-A(z)) - \int_0^x \gamma_1(t)dt} \qquad \text{..... (17)}$$

$$P_2(x,z) = P_2(0,z)\,e^{-d\lambda(1-A(z)) - \int_0^x \gamma_2(t)dt} \qquad \text{..... (18)}$$

$$D_1(x,z) = D_1(0,z)\,e^{-d\lambda(1-A(z)) - \int_0^x \gamma_1(t)dt} \qquad \text{..... (19)}$$

$$D_2(x,z) = D_2(0,z)\,e^{-d\lambda(1-A(z)) - \int_0^x \gamma_2(t)dt} \qquad \text{..... (20)}$$

Next we multiply equation (8) with appropriate powers of z, take summation over all possible values of n and using relations (7) and (17), after simplifying we obtain

$$zP_1(0,z) = (1-q)\mu_1\left[(1-r_1)\int_0^\infty P_1(x,z)\gamma_1(x)dx + (1-r_2)\int_0^\infty P_2(x,z)\gamma_2(x)dx\right] +$$

$$(1-q)\mu_1\left[\int_0^\infty D_1(x,z)\gamma_1(x)dx + \int_0^\infty D_2(x,z)\gamma_2(x)dx\right] + zd\mu_1\lambda(A(z)-1)E \qquad \text{..... (21)}$$

We perform the same operations on equations (9) – (11) and thus obtain

$$zP_2(0,z) = (1-q)\mu_2\left[(1-r_1)\int_0^\infty P_1(x,z)\gamma_1(x)dx + (1-r_2)\int_0^\infty P_2(x,z)\gamma_2(x)dx\right] +$$

$$(1-q)\mu_2\left[\int_0^\infty D_1(x,z)\gamma_1(x)dx + \int_0^\infty D_2(x,z)\gamma_2(x)dx\right] + zd\mu_2\lambda(A(z)-1)E \qquad \text{..... (22)}$$

$$D_{n,1}(0) = r_1\int_0^\infty P_1(x,z)\gamma_1(x)dx \qquad \text{..... (23)}$$

$$D_{n,2}(0) = r_2\int_0^\infty P_2(x,z)\gamma_2(x)dx \qquad \text{..... (24)}$$

Now multiply equations (17) and (19) by $\gamma_1(x)$ and equations (18) and (20) by $\gamma_2(x)$, integrate by parts w.r. to x and also use equation (2).

$$\int_0^\infty P_1(x,z)\gamma_1(x)dx = P_1(0,z)V_1^*(d(\lambda-\lambda A(z))) \qquad \text{..... (25)}$$

$$\int_0^\infty P_2(x,z)\gamma_2(x)dx = P_2(0,z)V_2^*(d(\lambda-\lambda A(z))) \qquad \text{..... (26)}$$

$$\int_0^\infty D_1(x,z)\gamma_1(x)dx = D_1(0,z)V_1^*(d(\lambda-\lambda A(z))) \qquad \text{..... (27)}$$

$$\int_0^\infty D_2(x,z)\gamma_2(x)dx = D_2(0,z)V_2^*(d(\lambda-\lambda A(z))) \qquad \text{..... (28)}$$

where $V_j^*(d(\lambda-\lambda A(z))) = \int_0^\infty e^{-d(\lambda-\lambda A(z))}dV_j(x)$ is the Laplace-stieltjesTransform of j[th] type of service j = 1, 2.

Now substitute relations (25) to (28) in equation (21) and (23), we get

$$zP_1(0,z) = (1-q)\,\mu_1\,[(1-r_1)\,P_1(0,z)\,V_1^*(d(\lambda-\lambda A(z))] + (1-r_2)\,P_2(0,z)$$
$$V_2^*\,(d\,(\lambda-\lambda\,A(z))] + (1-q)\,\mu_1\,[P_1(0,z)\,r_1\,[V_1^*(d(\lambda-\lambda A(z)))]^2 +$$
$$P_2(0,z)r_2\,V_2^*\,(d\,(\lambda-\lambda\,A(z)))]^2] + z\lambda d\mu_1(A(z)-1))\,E \qquad \text{.....(29)}$$

$$zP_2(0,z) = (1-q)\,\mu_2\,[(1-r_1)\,P_1(0,z)\,V_1^*(d(\lambda-\lambda A(z))) + (1-r_2)\,P_2(0,z)$$
$$V_2^*\,(d\,(\lambda-\lambda\,A(z)))] + (1-q)\,\mu_2\,[P_1(0,z)\,r_1\,[V_1^*(d(\lambda-\lambda A(z)))]^2 +$$
$$P_2(0,z)r_2\,V_2^*\,(d\,(\lambda-\lambda\,A(z))]^2] + z\lambda d\mu_2(A(z)-1))\,E \qquad \text{.....(30)}$$

Solving (29) and (30) we get

$$P_1(0,z) = \frac{\lambda z\mu_1 d(1-A(z))E}{B(z)} \qquad \text{..... (31)}$$

$$P_2(0,z) = \frac{\lambda z\mu_2 d(1-A(z))E}{B(z)} \qquad \text{..... (32)}$$

$$\text{(ie)}D_1(0,z) = \frac{r_1\lambda z\mu_1 d(1-A(z)EV_1^*(d(\lambda-\lambda A(z)))}{B(z)} \qquad \text{..... (33)}$$

$$D_2(0,z) = \frac{r_2\lambda z\mu_2 d(1-A(z))EV_2^*(d(\lambda-\lambda A(z))}{B(z)} \qquad \text{..... (34)}$$

where B(z) is given by

$B(z) = (1 - q)\,[\mu_2 r_2 V_2^*\,(d(\lambda - \lambda\, A(z)))^2 + \mu_1 r_1 V_1^*(d(\lambda - \lambda A(z)))^2 + \mu_1(1 - r_1)$
$\quad V_1^*\,(d\,(\lambda - \lambda\, A(z))) - \mu_2\,(1 - r_2)\,V_2^*\,(d\,(\lambda - \lambda\, A(z)))] - z$

Now Define $P_Q(z)$ as the probability generating function of the queue size irrespective of the type of service the server is providing,

$P_Q(z) = P_1(z) + P_2(z) + D_1(z) + D_2(z)$

To determine the unknown probability E by using the relation of the normalizing condition $E + P_1(1) + P_2(1) + D_1(1) + D_2(1) = 1$

$$P_1(1) = \lim_{z\to 1} P_1(z)$$

$$= \frac{\mu_1 d\lambda E(I)E(w_1)E}{(1-q)\begin{bmatrix}\mu_2 r_2 E(w_2)d\lambda E(I) + \mu_1 r_1 E(w_1)d\lambda E(I) + \\ \mu_1(1-r_1)E(w_1)d\lambda E(I) - \mu_2(1-r_2)E(w_2)d\lambda E(I)\end{bmatrix} - 1}$$

$$P_1(1) = \frac{\mu_1 d\lambda E(I)E(w_1)E}{(1-q)d\lambda E(I)\begin{bmatrix}\mu_1 r_1 E(w_1) + \mu_2 r_2 E(w_2) + \mu_1(1-r_1) \\ E(w_1) - \mu_2(1-r_2)E(w_2)\end{bmatrix} - 1} \qquad \text{..... (35)}$$

$$P_2(1) = \lim_{z\to 1} P_2(z)$$

$$= \frac{\mu_2 d\lambda E(I)E(w_2)E}{(1-q)d\lambda E(I)\begin{bmatrix}\mu_1 r_1 E(w_1) + \mu_2 r_2 E(w_2) + \mu_1(1-r_1) \\ E(w_1) - \mu_2(1-r_2)E(w_2)\end{bmatrix} - 1} \qquad \text{..... (36)}$$

$$D_1(1) = \lim_{z\to 1} D_1(0,z)$$

$$= \frac{r_1\mu_1 d\lambda E(I)E(w_1)E}{(1-q)d\lambda E(I)\begin{bmatrix}\mu_1 r_1 E(w_1) + \mu_2 r_2 E(w_2) + \mu_1(1-r_1) \\ E(w_1) - \mu_2(1-r_2)E(w_2)\end{bmatrix} - 1} \qquad \text{..... (37)}$$

$$D_2(1) = \lim_{z\to 1} D_2(0,z)$$

$$= \frac{r_2\mu_2 d\lambda E(I)E(w_2)E}{(1-q)d\lambda E(I)\begin{bmatrix}\mu_1 r_1 E(w_1) + \mu_2 r_2 E(w_2) + \mu_1(1-r_1) \\ E(w_1) - \mu_2(1-r_2)E(w_2)\end{bmatrix} - 1} \qquad \text{..... (38)}$$

where E(I) is the mean size of batch of arriving customer, $E(w_1)$, $E(w_2)$ are the mean service time of type 1 and type 2 server respectively and $V^*(0) = 1$.
The right hand side of equation (35) – (38) gives the steady state probability that the server is busy providing type 1 service, type 2 services, repeating service of type 1 and type 2 respectively.

Now adding equations (35) – (38) we get

$$P_Q(1) = \frac{d\lambda E(I)\left[\mu_1 E(w_1) + \mu_2 E(w_2) + r_1\mu_1 E(w_1) + r_2\mu_2 E(w_2)\right]E}{(1-q)\, d\lambda E(I)\begin{bmatrix}\mu_1 r_1 E(w_1) + \mu_2 r_2 E(w_2) + \\ \mu_1(1-r_1)E(w_1) - \mu_2(1-r_2)E(w_2)\end{bmatrix} - 1} \quad \text{..... (39)}$$

Which gives the steady state probability that the server is busy.

Now using the normalizing condition E + $P_Q(1)$ = 1, we get

$$= \frac{1-(1-q)d\lambda E(I)\begin{bmatrix}\mu_1\gamma_1 E(w_1) + \mu_2\gamma_2 E(w_2) + \mu_1(1-\gamma_1)E(w_1) \\ -\mu_2(1-\gamma_2)E(w_2)\end{bmatrix}}{1-d\lambda E(I)\begin{bmatrix}(1-q)[\mu_1\gamma_1 E(w_1) + \mu_2\gamma_2 E(w_2) + \mu_1(1-\gamma_1)E(w_1) \\ -\mu_2(1-\gamma_2)E(w_2))\mu_1 E(w_1) + \mu_2 E(w_2) + \gamma_1\mu_1 E(w_1) \\ +\mu_2\gamma_2 E(w_2)]\end{bmatrix}} \quad \text{..... (40)}$$

The utilization factor ρ of the system can be calculated using ρ = 1 – E. Let $P_s(z)$ denote the probability generating function of the system and it can calculated as $P_s(z) = P_1(z) + P_2(z) + D_1(z) + D_2(z) + E$

$$P_s(z) = \frac{E\begin{bmatrix} z\mu_1(1-V*(d(\lambda-\lambda A(z))) + z\mu_2(1-V_2^*(d(\lambda-\lambda A(z))) + \\ r_1 z\mu_1 V_1^*[d(\lambda-\lambda A(z)]\ [1-V_1^*(d(\lambda-\lambda A(z)] + \\ r_2 z\mu_2 V_2^*[d(\lambda-\lambda A(z))]\ [1-V_1^*(d(\lambda-\lambda A(z))] + \\ (1-q)[\mu_2 r_2 V_2^*(d(\lambda-\lambda A(z))^2 + \mu_1 r_1 V_1^*(d(\lambda-\lambda A(z)))^2 + \\ \mu_1(1-r_1)V_1^*(d(\lambda-\lambda A(z))) - \mu_2(1-r_2)V_2^*(d(\lambda-\lambda A(z)))] - z\end{bmatrix}}{B(z)} \quad \text{..... (41)}$$

The Average Queue Size

Let L_q denote the expected number of patients in the Queue at random epoch.

Then $L_q = \lim_{z\to 1} \frac{d}{dz} P_Q(z)$

Also we the formula L = L_q + ρand obtain the steady- state average size of the queue system.

Particular case
Case (i)

Re- service in a queue with Batch arrival and two types of General Heterogeneous service. Here we consider the situation where there is no balking. (ie) all the customers join the system, in this case take d = 1, then $P_s(z)$ becomes

(41) ⇒

$$P_s(z) = \frac{E\begin{bmatrix} z\mu_1(1-V*(\lambda-\lambda A(z))) + z\mu_2(1-V_2^*(\lambda-\lambda A(z))) + \\ r_1 z\mu_1 V_1^*(\lambda-\lambda A(z)))\ [1-V_1^*(\lambda-\lambda A(z))) + \\ r_2 z\mu_2 V_2^*[\lambda-\lambda A(z))]\ [1-V_2^*(\lambda-\lambda A(z))] + \\ (1-q)[\mu_2 r_2 V_2^*(\lambda-\lambda A(z)))^2 + \mu_1 r_1 V_1^*(\lambda-\lambda A(z)))^2 + \\ \mu_1(1-r_1)V_1^*(\lambda-\lambda A(z))) - \mu_2(1-r_2)V_2^*(\lambda-\lambda A(z)))] - z\end{bmatrix}}{\begin{matrix}(1-q)[\mu_2 r_2 V_2^*(\lambda-\lambda A(z)))^2 + \mu_1 r_1 V_1^*(\lambda-\lambda A(z))^2 + \\ \mu_1(1-r_1)V_1^*(\lambda-\lambda A(z))) - \mu_2(1-r_2)V_2^*(\lambda-\lambda A(z))) - z\end{matrix}}$$

Also

$$E = \frac{1-(1-q)\lambda E(I)[\mu_1 r_1 E(w_1)+\mu_2 r_2 E(w_2)+\mu_1(1-r_1)E(w_1) - \mu_2(1-r_2)E(w_2)}{1-\lambda E(I)\begin{bmatrix}(1-q)\,[\mu_1 r_1 E(w_1)+\mu_2 r_2 E(w_2)+\mu_1(1-r_1)E(w_1) - \\ \mu_2(1-r_2)E(w_2)] + \mu_1 E(w_1)+\mu_2 E(w_2)+r_1\mu_1 E(w_1) \\ +\mu_2 r_2 E(w_2)\end{bmatrix}}$$

case (ii)

In this case assume there is no re-service then $r_1 = r_2 = 0$, Queue size distribution reduces to

$$= \frac{\begin{gathered}E[z\mu_1(1-V_1^*(d(\lambda-\lambda A(z)))+z\mu_2(1-V_2^*(d(\lambda-\lambda A(z)))-\lambda A(z)] \\ \left[1-V_1^*(d(\lambda-\lambda A(z)))\right]+(1\text{-}q)\,[\mu_1 V_1^*(d(\lambda - \lambda A(z)) - \\ \mu_2 V_2^*\,(d\,(\lambda-\lambda A(z)))-z]\end{gathered}}{(1\text{-}q)\,[\mu_1 V_1^*(d(\lambda - \lambda A(z)) - \mu_2 V_2^*\,(d(\lambda-\lambda A(z)))]-z}$$

$$E = \frac{1-(1-q)d\lambda E(I)[\mu_1 E(w_1)-\mu_2 E(w_2)]}{1-d\lambda E(I)[(1-q)[\mu_1 E(w_1)-\mu_2 E(w_2)]+\mu_1 E(w_1)+\mu_2 E(w_2)]}$$

case (iii)

In this case assume there is no optional re-service and no balking (ie) $r_1=r_2=0$ and $d = 1$, then

$$P_s(z) = \frac{\begin{gathered}E[z\mu_1(1-V_1^*(\lambda-\lambda A(z))+z\mu_2(1-V_2^*(\lambda-\lambda A(z))-\lambda A(z)] \\ \left[1-V_1^*(\lambda-\lambda A(z))\right]+(1\text{-}q)\,[\ \mu_1 V_1^*(\lambda - \lambda A(z)) - \\ \mu_2 V_2^*\,(\lambda-\lambda A(z))-z]\end{gathered}}{(1\text{-}q)\,[\mu_1 V_1^*(\lambda - \lambda A(z)) - \mu_2 V_2^*\,(\lambda-\lambda A(z))]-z}$$

$$E = \frac{1-(1-q)d\lambda E(I)[\mu_1 E(w_1)-\mu_2 E(w_2)]}{1-\lambda E(I)[(1-q)[\mu_1 E(w_1)-\mu_2 E(w_2)]+\mu_1 E(w_1)+\mu_2 E(w_2)]}$$

CONCLUSION

In this paper, a balking and re-service in a bulk queue with two types of heterogeneous service is analyzed. We introduced the concept of optional re-service. The system performance measure obtained explicitly. Also we discussed special cases .

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