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Application of Arithmetic Operations on Icosagonal (ICO) Fuzzy Numbers and its Utility

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Abstract- **A research focuses on real life problems, which plays an important role in making life comfortable. In this paper, we discuss about Icosagonal (ICO) fuzzy numbers with their membership function. Also arithmetic operations on α- cut of Icosagonal (ICO) fuzzy numbers are also examined. The main objective of this paper is to introduce arithmetic operations like addition, subtraction, multiplication, division on Icosagonal (ICO) fuzzy numbers to validate the new result. To validate the results few examples are also proposed.**

Keywords—Fuzzy Set, membership functions, α- cut of Icosagonal (ICO) fuzzy numbers, Arithmetic operations, Ranking of fuzzy numbers.

I. Introduction

It is very easy to deal with single value function but when the data is not available in such a pattern according to real word problems then this becomes a challenge in decision making of such issues. To deal with multivalve data issues, in 1965 Zadeh introduced fuzzy theory. It gives simple means to deal with such problems in which source of information or the input data is in form of fuzzy numbers. This can be applied to variety of field's decision making, control system, optimization problems. The fuzzy number can be defined as fuzzy subset of real line by Dubosis, H. Parde [1]. A fuzzy number is a quantity whose value is precise rather than exact. Different types of fuzzy numbers are described in literature such as triangular, trapezoidal, pentagonal, hexagonal, etc. The membership can be applied to variety of tasks and having wide applications. This Paper is organized as follows: In Section 2 basic definitions of an ICO fuzzy numbers and some operations on it. In Section 3 Alpha cuts of ICO fuzzy numbers are described. Anew arithmetic operation on Alpha cut is defined in Section 4. In next section, numerical example is given. Finally, Section 6, conclusion is included.

II. Preliminaries

Definition 2.1 Fuzzy set:
Let X ≠ { }. A fuzzy set 'A' in X is characterized by its membership function μA(x), in which image of each value from domain mapped into the unit interval [0, 1]. The value of μA(x) represents greatest membership of x ε μA(x). The general representation for a fuzzy set is
A = {x, μA(x); x belongs to X}

$\mu A(x)$: $A \rightarrow [0, 1]$

Definition 2.2 Fuzzy number: A fuzzy number "A" is a convex normalized fuzzy set on the real number line, such that: (i) There exit at least one x ε R with the condition that μA(x) = 1, (ii) μA(x) is piece- wise continuous.

Definition 2.3 Crisp set: A crisp is special case of fuzzy set in which the membership function takes only two values 0 and 1.

Definition 2.4 Icosagonal Fuzzy Number (ICO): The Icosagonal fuzzy number is denoted by $\tilde{A}$ (ICO) and its membership function is given below. For this we consider 20 real numbers, such that
$a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_6 \leq a_7 \leq a_8 \leq a_9 \leq a_{10} \leq a_{11} \leq a_{12} \leq a_{13} \leq a_{14} \leq a_{15} \leq a_{16} \leq a_{17} \leq a_{18} \leq a_{19}$ $a \leq_{20}$

Definition 2.5 α–cut: The α-cut of the fuzzy set A of universe of discourse X is defined as A_α = {x ε X: μA(x) ≥ α: α ε [0, 1]}

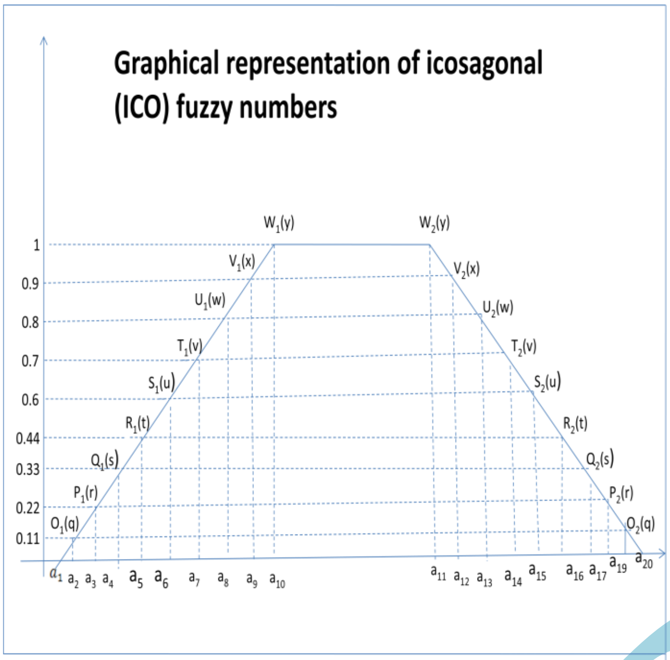

Figure 1: Graphical Representation of Icosagonal (ICO) Fuzzy Numbers

Equation: Calculating Fuzzy value of ICO Numbers

$$\tilde{A}_{ICFN} = \begin{cases} 0 & \text{for } x < a_1 \\ \frac{1}{9}\frac{(x-a_1)}{(a_2-a_1)} & \text{for } a_1 \le x \le a_2 \\ \frac{1}{9}+\frac{1}{9}\frac{(x-a_2)}{(a_3-a_2)} & \text{for } a_2 \le x \le a_3 \\ \frac{2}{9}+\frac{1}{9}\frac{(x-a_3)}{(a_4-a_3)} & \text{for } a_3 \le x \le a_4 \\ \frac{3}{9}+\frac{1}{9}\frac{(x-a_4)}{(a_5-a_4)} & \text{for } a_4 \le x \le a_5 \\ \frac{4}{9}+\frac{1}{9}\frac{(x-a_5)}{(a_6-a_5)} & \text{for } a_5 \le x \le a_6 \\ \frac{5}{9}+\frac{1}{9}\frac{(x-a_6)}{(a_7-a_6)} & \text{for } a_6 \le x \le a_7 \\ \frac{6}{9}+\frac{1}{9}\frac{(x-a_7)}{(a_8-a_7)} & \text{for } a_7 \le x \le a_8 \\ \frac{7}{9}+\frac{1}{9}\frac{(x-a_8)}{(a_9-a_8)} & \text{for } a_8 \le x \le a_9 \\ \frac{8}{9}+\frac{1}{9}\frac{(x-a_9)}{(a_{10}-a_9)} & \text{for } a_8 \le x \le a_9 \end{cases}$$

Equation: Calculating Fuzzy value of ICO Numbers

$$\tilde{A}_{ICO} = \begin{cases} 1 & \text{for } a_{10} \le x \le a_{11} \\ 1-\frac{1}{9}\frac{(x-a_{11})}{(a_{12}-a_{11})} & \text{for } a_{11} \le x \le a_{12} \\ \frac{8}{9}-\frac{1}{9}\frac{(x-a_{12})}{(a_{13}-a_{12})} & \text{for } a_{12} \le x \le a_{13} \\ \frac{7}{9}-\frac{1}{9}\frac{(x-a_{13})}{(a_{14}-a_{13})} & \text{for } a_{13} \le x \le a_{14} \\ \frac{6}{9}-\frac{1}{9}\frac{(x-a_{14})}{(a_{15}-a_{14})} & \text{for } a_{14} \le x \le a_{15} \\ \frac{5}{9}-\frac{1}{9}\frac{(x-a_{15})}{(a_{16}-a_{15})} & \text{for } a_{15} \le x \le a_{16} \\ \frac{4}{9}-\frac{1}{9}\frac{(x-a_{16})}{(a_{17}-a_{16})} & \text{for } a_{16} \le x \le a_{17} \\ \frac{3}{9}-\frac{1}{9}\frac{(x-a_{17})}{(a_{18}-a_{17})} & \text{for } a_{17} \le x \le a_{18} \\ \frac{2}{9}-\frac{1}{9}\frac{(x-a_{18})}{(a_{19}-a_{18})} & \text{for } a_{18} \le x \le a_{19} \\ \frac{1}{9}\frac{(a_{20}-x)}{(a_{20}-a_{19})} & \text{for } a_{19} \le x \le a_{20} \\ 0 & \text{for } x \ge a_{20} \end{cases}$$

III. Ranking Function

The ranking function Ř: F(r) →R, which maps each fuzzy number to real line, where the natural order exists which is defined by Yager 1986 i.e. Let

$A = \{a_1, a_2, a_3, a_4\ a_5, a_6, a_7, a_8, a_9\ a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}\}$ and

$B = \{[b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16}, b_{17}, b_{18}, b_{19}, b_{20}]$ are two Icosagonal fuzzy numbers then

$Ř(A) = [\{ a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10} + a_{11} + a_{12} + a_{13} + a_{14} + a_{15} + a_{16} + a_{17} + a_{18} + a_{19} + a_{20}\}/20]$

$Ř(B) = [\{ b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7 + b_8 + b_9 + b_{10} + b_{11} + b_{12} + b_{13} + b_{14} + b_{15} + b_{16} + b_{17} + b_{18} + b_{19} + b_{20} \}/20]$

Results on ranking -

Also orders on F(R) are defined as:

A ≥ B if and only if Ř (A) ≥ Ř (B)

A ≤ B if and only if Ř (A) ≤ Ř (B)

A = B if and only if Ř (A) = Ř (B)

IV. Arithmetic Operations on ICO Fuzzy Numbers

If $A = \{a_1, a_2, a_3, a_4\ a_5, a_6, a_7, a_8, a_9\ a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}\}$ and

B = $\{b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16}, b_{17}, b_{18}, b_{19}, b_{20}\}$ are two Icosagonal fuzzy numbers then

Addition

A+B = $(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7, a_8 + b_8, a_9 + b_9, a_{10} + b_{10}, a_{11} + b_{11}, a_{12} + b_{12}, a_{13} + b_{13}, a_{14} + b_{14}, a_{15} + b_{15}, a_{16} + b_{16}, a_{17} + b_{17}, a_{18} + b_{18}, a_{19} + b_{19}, a_{20} + b_{20})$

Subtraction

A-B = $(a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4, a_5 - b_5, a_6 - b_6, a_7 - b_7, a_8 - b_8, a_9 - b_9, a_{10} - b_{10}, a_{11} - b_{11}, a_{12} - b_{12}, a_{13} - b_{13}, a_{14} - b_{14}, a_{15} - b_{15}, a_{16} - b_{16}, a_{17} - b_{17}, a_{18} - b_{18}, a_{19} - b_{19}, a_{20} - b_{20})$

Multiplication

let $\pi_b = (b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7 + b_8 + b_9 + b_{10} + b_{11} + b_{12} + b_{13} + b_{14} + b_{15} + b_{16} + b_{17} + b_{18} + b_{19} + b_{20})$

then

$A^* B = (a_1\pi_b/20, a_2\pi_b/20, a_3\pi_b/20, a_4\pi_b/20, a_5\pi_b/20, a_6\pi_b/20, a_7\pi_b/20, a_8\pi_b/20, a_9\pi_b/20, a_{10}\pi_b/20, a_{11}\pi_b/20, a_{12}\pi_b/20, a_{13}\pi_b/20, a_{14}\pi_b/20, a_{15}\pi_b/20, a_{16}\pi_b/20, a_{17}\pi_b/20, a_{18}\pi_b/20, a1\pi b/20, a_{20}\pi_b/20\}$

Division:

Let $\pi_b = (b1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7 + b_8 + b_9 + b_{10} + b_{11} + b_{12} + b_{13} + b_{14} + b_{15} + b_{16} + b_{17} + b_{18} + b_{19} + b_{20})$

then

A/B = $(a_1 20/\pi_b, a_2 20/\pi_b, a_9 20/\pi_b, a_{10} 20/\pi_b, a_{11} 20/\pi_b, a_{12} 20/\pi_b, a_{13} 20/\pi_b, a_{14} 20/\pi_b, a_{15} 20/\pi_b, a_{16} 20/\pi_b, a_{17} 20/\pi_b, a_{18} 20/\pi_b, a_{19} 20/\pi_b, a_{20}\ 20/\pi_b)$

V. THE α --CUT OF ICO FUZZY NUMBERS

The crisp set A is called alpha cut is defined as-

$A_\alpha = \{x \varepsilon X: \mu A(x) \geq \alpha: \alpha \varepsilon [0, 1]\}$

$$A_\alpha = \begin{cases} [d_l(\alpha), d_u(\alpha)] & \text{for } \alpha \varepsilon [0.00, 0.11] \\ [e_l(\alpha), e_u(\alpha)] & \text{for } \alpha \varepsilon [0.11, 0.22] \\ [f_l(\alpha), f_u(\alpha)] & \text{for } \alpha \varepsilon [0.22, 0.33] \\ [g_l(\alpha), g_u(\alpha)] & \text{for } \alpha \varepsilon [0.33, 0.44] \\ [h_l(\alpha), h_u(\alpha)] & \text{for } \alpha \varepsilon [0.44, 0.55] \\ [i_l(\alpha), i_u(\alpha)] & \text{for } \alpha \varepsilon [0.55, 0.66] \\ [j_l(\alpha), j_u(\alpha)] & \text{for } \alpha \varepsilon [0.66, 0.77] \\ [k_l(\alpha), k_u(\alpha)] & \text{for } \alpha \varepsilon [0.77, 0.88] \\ [l_l(\alpha), l_u(\alpha)] & \text{for } \alpha \varepsilon [0.88, 0.99] \end{cases}$$

VI. OPERATIONS ON VARIABLES

A- New α-cut operation: A The distribution of interval A_α, for all α ε [0, 1] is obtained as follows:

Consider $[d_l(\alpha), d_u(\alpha)] = \frac{1}{9}\frac{(x-a_1)}{(a_2-a_2)}$ and $\frac{1}{9}\frac{(a_{20}-x)}{(a_{20}-a_{19})}$

on simplification, we obtain

$[d_l(\alpha), d_u(\alpha)] = \{a_1 + 9\alpha(a_2 - a_1), a_{20} + 9\alpha(a_{20} - a_{19})\}$

Consider $[e_u(\alpha), e_u(\alpha)] = \frac{1}{9}+\frac{1}{9}\frac{(x-a_2)}{(a_3-a_2)}$ and $\frac{2}{9}-\frac{1}{9}\frac{(x-a_{18})}{(a_{19}-a_{18})}$

on simplification, we obtain

$[e_l(\alpha), d_u(\alpha)] = \{a_2 + (9\alpha - 1)(a_3 - a_2), a_{19} + (2-9\alpha)(a_{19} - a_{18})\}$

Consider $[f_l(\alpha), f_u(\alpha)] = \frac{2}{9}+\frac{1}{9}\frac{(x-a_3)}{(a_5-a_3)}$ and $\frac{3}{9}-\frac{1}{9}\frac{(x-a_{17})}{(a_{18}-a_{17})}$

on simplification, we obtain

$[f_l(\alpha), f_u(\alpha)] = \{a_3 + (9\alpha - 2)(a_4 - a_3), a_{18} + (3-9\alpha)(a_{18} - a_{17})\}$

Consider $[g_l(\alpha), g_u(\alpha)] = \frac{3}{9}+\frac{1}{9}\frac{(x-a_4)}{(a_5-a_4)}$ and $\frac{4}{9}-\frac{1}{9}\frac{(x-a_{16})}{(a_{17}-a_{16})}$

on simplification, we obtain

$[g_l(\alpha), g_u(\alpha)] = \{a_4 + (9\alpha - 3)(a_5 - a_4), a_{17} + (4-9\alpha)(a_{17} - a_{16})\}$

Consider $[h_l(\alpha), h_u(\alpha)] = \frac{4}{9}+\frac{1}{9}\frac{(x-a_5)}{(a_6-a_5)}$ and $\frac{5}{9}-\frac{1}{9}\frac{(x-a_{15})}{(a_{16}-a_{15})}$

on simplification, we obtain

$[h_l(\alpha), h_u(\alpha)] = \{a_5 + (9\alpha - 4)(a_6 - a_5), a_{16} + (5-9\alpha)(a_{16} - a_{15})\}$

Consider $[i_l(\alpha), i_u(\alpha)] = \frac{5}{9}+\frac{1}{9}\frac{(x-a_6)}{(a_7-a_6)}$ and $\frac{6}{9}-\frac{1}{9}\frac{(x-a_{14})}{(a_{15}-a_{14})}$

on simplification, we obtain

$[i_l(\alpha), i_u(\alpha)] = \{a_6 + (9\alpha - 5)(a_7 - a_6), a_{15} + (6-9\alpha)(a_{15} - a_{14})\}$

Consider $[j_l(\alpha), j_u(\alpha)] = \frac{6}{9}+\frac{1}{9}\frac{(x-a_7)}{(a_8-a_7)}$ and $\frac{7}{9}-\frac{1}{9}\frac{(x-a_{13})}{(a_{14}-a_{13})}$

on simplification, we obtain

$[j_l(\alpha), j_u(\alpha)] = \{a_7 + (9\alpha - 6)(a_8 - a_7), a_{14} + (7-9\alpha)(a_{14} - a_{13})\}$

Consider $[l_l(\alpha), l_u(\alpha)] = \frac{8}{9}+\frac{1}{9}\frac{(x-a_9)}{(a_{10}-a_9)}$ and $1-\frac{1}{9}\frac{(x-a_{11})}{(a_{12}-a_{11})}$

on simplification, we obtain

$[l_l(\alpha), l_u(\alpha)] = \{a_9 + (9\alpha - 8)(a_{10} - a_9), a_{12} + (9-9\alpha)(a_{12} - a_{11})\}$

Consider $[k_l(\alpha), k_u(\alpha)] = \frac{7}{9}+\frac{1}{9}\frac{(x-a_8)}{(a_9-a_8)}$ and $\frac{8}{9}-\frac{1}{9}\frac{(x-a_{12})}{(a_{13}-a_{12})}$

on simplification, we obtain

$[k_l(\alpha), k_u(\alpha)] = \{a_8 + (9\alpha - 7)(a_9 - a_8), a_{13} + (8-9\alpha)(a_{13} - a_{12})\}$

All above are the distributions of interval A_α, for the domain Value [0, 1].

B- New arithmetic operations on ICO fuzzy numbers using α–cut: The arithmetic operations on α-cut of ICO fuzzy number is given below: let

$A = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}\}$

$B = \{b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16}, b_{17}, b_{18}, b_{19}, b_{20}\}$

$$A_\alpha = \begin{cases} a_1 + 9\alpha(a_2 - a_1), & a_{20} - 9\alpha(a_{20} - a_{19}) & \text{for } \alpha \,\varepsilon\, [0, 0.11] \\ a_2 + (9\alpha - 1)(a_3 - a_2), & a_{19} . (2 - 9\alpha)(a_{19} - a_{18}) & \text{for } \alpha \,\varepsilon\, [0.11, 0.22] \\ a_3 + (9\alpha - 2)(a_4 - a_3), & a_{18} - (3 - 9\alpha)(a_{18} - a_{17}) & \text{for } \alpha \,\varepsilon\, [0.22, 0.33] \\ a_4 + (9\alpha - 3)(a_5 - a_4), & a_{17} - (4 - 9\alpha)(a_{17} - a_{16}) & \text{for } \alpha \,\varepsilon\, [0.33, 0.44] \\ a_5 + (9\alpha - 4)(a_6 - a_5), & a_{16} - (5 - 9\alpha)(a_{16} - a_{15}) & \text{for } \alpha \,\varepsilon\, [0.44, 0.55] \\ a_6 + (9\alpha - 5)(a_7 - a_6), & a_{15} - (6 - 9\alpha)(a_{15} - a_{14}) & \text{for } \alpha \,\varepsilon\, [0.55, 0.66] \\ a_7 + (9\alpha - 6)(a_8 - a_7), & a_{14} - (7 - 9\alpha)(a_{14} - a_{13}) & \text{for } \alpha \,\varepsilon\, [0.66, 0.77] \\ a_8 + (9\alpha - 7)(a_9 - a_8), & a_{13} - (8 - 9\alpha)(a_{13} - a_{12}) & \text{for } \alpha \,\varepsilon\, [0.77, 0.88] \\ a_9 + (9\alpha - 8)(a_{10} - a_9), & a_{12} - (9 - 9\alpha)(a_{12} - a_{11}) & \text{for } \alpha \,\varepsilon\, [0.88, 0.99] \end{cases}$$

$$B_\alpha = \begin{cases} b_1 + 9\alpha(b_2 - b_1), & b_{20} - 9\alpha(b_{20} - b_{19}) & \text{for } \alpha \,\varepsilon\, [0, 0.11] \\ b_2 + (9\alpha - 1)(b_3 - b_2), & b_{19} . (2 - 9\alpha)(b_{19} - b_{18}) & \text{for } \alpha \,\varepsilon\, [0.11, 0.22] \\ b_3 + (9\alpha - 2)(b_4 - b_3), & b_{18} - (3 - 9\alpha)(b_{18} - b_{17}) & \text{for } \alpha \,\varepsilon\, [0.22, 0.33] \\ b_4 + (9\alpha - 3)(b_5 - a_4), & b_{17} - (4 - 9\alpha)(b_{17} - b_{16}) & \text{for } \alpha \,\varepsilon\, [0.33, 0.44] \\ b_5 + (9\alpha - 4)(b_6 - b_5), & b_{16} - (5 - 9\alpha)(b_{16} - b_{15}) & \text{for } \alpha \,\varepsilon\, [0.44, 0.55] \\ b_6 + (9\alpha - 5)(b_7 - b_6), & b_{15} - (6 - 9\alpha)(b_{15} - b_{14}) & \text{for } \alpha \,\varepsilon\, [0.55, 0.66] \\ b_7 + (9\alpha - 6)(b_8 - b_7), & b_{14} - (7 - 9\alpha)(b_{14} - b_{13}) & \text{for } \alpha \,\varepsilon\, [0.66, 0.77] \\ b_8 + (9\alpha - 7)(b_9 - b_8), & b_{13} - (8 - 9\alpha)(b_{13} - b_{12}) & \text{for } \alpha \,\varepsilon\, [0.77, 0.88] \\ b_9 + (9\alpha - 8)(b_{10} - b_9), & b_{12} - (9 - 9\alpha)(b_{12} - b_{11}) & \text{for } \alpha \,\varepsilon\, [0.88, 0.99] \end{cases}$$

C- Numerical example:

Let A_α = (2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21)
B_α = (3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41)

$A_\alpha + B_\alpha$ = (2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21)

(+)

(3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41)

$A_\alpha + B_\alpha$ = (5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41, 44, 47, 50, 53, 56, 59, 62)

By new arithmetic operations on Icosagonal fuzzy number, we have

$$A_\alpha + B_\alpha = \begin{cases} [5 + 27\alpha,\ 62 - 27\alpha] & \text{for } \alpha \,\varepsilon\, [0, 0.11] \\ [5 + 27\alpha,\ 53 + 27\alpha] & \text{for } \alpha \,\varepsilon\, [0.11, 0.22] \\ [5 + 27\alpha,\ 47 + 27\alpha] & \text{for } \alpha \,\varepsilon\, [0.22, 0.33] \\ [5 + 27\alpha,\ 41 + 27\alpha] & \text{for } \alpha \,\varepsilon\, [0.33, 0.44] \\ [5 + 27\alpha,\ 35 + 27\alpha] & \text{for } \alpha \,\varepsilon\, [0.44, 0.55] \\ [5 + 27\alpha,\ 29 + 27\alpha] & \text{for } \alpha \,\varepsilon\, [0.55, 0.66] \\ [5 + 27\alpha,\ 23 + 27\alpha] & \text{for } \alpha \,\varepsilon\, [0.66, 0.77] \\ [5 + 27\alpha,\ 17 + 27\alpha] & \text{for } \alpha \,\varepsilon\, [0.77, 0.88] \\ [5 + 27\alpha,\ 11 + 27\alpha] & \text{for } \alpha \,\varepsilon\, [0.88, 0.99] \end{cases}$$

When

$\alpha = 0.00$,	A_0	(+)	B_0	=	(5, 62)
$\alpha = 0.11$,	$A_{0.11}$	(+)	$B_{0.11}$	=	(8, 59)
$\alpha = 0.22$,	$A_{0.22}$	(+)	$B_{0.22}$	=	(11, 53)
$\alpha = 0.33$,	$A_{0.33}$	(+)	$B_{0.33}$	=	(14, 50)
$\alpha = 0.44$,	$A_{0.44}$	(+)	$B_{0.44}$	=	(17, 47)
$\alpha = 0.55$,	$A_{0.55}$	(+)	$B_{0.55}$	=	(20, 44)
$\alpha = 0.66$,	$A_{0.66}$	(+)	$B_{0.66}$	=	(23, 41)
$\alpha = 0.77$,	$A_{0.77}$	(+)	$B_{0.77}$	=	(26, 38)
$\alpha = 0.88$,	$A_{0.88}$	(+)	$B_{0.88}$	=	(29, 35)
$\alpha = 1.0$	A_1	(+)	B_1	=	(32, 56)

Hence

A_α (+) B_α = (5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41, 44, 47, 50, 53, 56, 59, 62)

Hence all the points coincide with the sum of the two Icosagonal fuzzy numbers. A similar process can be followed for difference of Icosagonal fuzzy numbers. We can say that the positive side of this result is if the data is given in vague form then very near to optimal answer can be calculated by using this method. The disadvantages as we know the calculated value is approximate of actual value of Icosagonal (ICO) fuzzy number therefore it gives a way to research further with new idea. Overall the we can say as many life problems or situations has vague data or we can say about data value then by using this concept a vague or ambiguous data can be converted into single-valued number data which helps in solving many real life related complex problems by giving a proper shape value to data.

VII. Conclusion

In this paper, a new Icosagonal (ICO) fuzzy number is introduced and used to study the arithmetic operations on those fuzzy numbers. We define α -cut of Icosagonal fuzzy number and also discuss the effect of arithmetic operation on Icosagonal fuzzy number. Also an example is given for showing the effect of arithmetic operation on ICO number. Alpha cut of ICO fuzzy numbers is also studied and related operations are presented.

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