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Common fixed point theorem for Chatterjea-Type Contractive mapping in Complex Valued Metric Space

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Abstract: Study of Multivalued contraction mapping was initiated by Nadler[1] and Markin[2]. There are so many research papers in which multiple common fixed point theorem results are proved by authors[3-10].This paper deals with some common fixed point theorems which are established for multi valued mapping in complex valued metric space as Azam et al.(numer.Funct.anal.Optim.33(5):590-600,2012) introduced the notion of complex valued metric space and proved some common fixed point theorems in the context of complex valued metric space.

Keywords: multi-valued mapping, common fixed point, complex valued metric space.

I. PRELIMINARIES

Let us recall a natural relation on ℂ,for $z_1,z_2\in$ ℂ, define a partial order $\precsim$ on ℂ as follows;

$z_1\precsim z_2$ iff Re(z1) ≤Re(z2), Im(z_1) ≤Im(z_2)

it follows that

$z_1\precsim z_2$

if one of the following conditions is satisfied:

i. Re(z1)=Re(z2), Im(z_1)<Im(z_2)
ii. Re(z1)<Re(z2), Im(z_1)=Im(z_2)
iii. Re(z1)<Re(z2), Im(z_1)<Im(z_2)
iv. Re(z1)=Re(z2), Im(z_1)=Im(z_2)

In particular, we will write $z_1\precnsim z_2$ if $z_1\neq z_2$ and one the above conditions is not satisfied and we will write $z_1\prec z_2$ if only iii is satisfied. Note that $0\precsim z_1\precnsim z_2\Rightarrow|z_1|<|z_2|$,

$z_1\precsim z_2$, $z_1\prec z_2\Rightarrow z_1\prec z_3$

Definition 1 let X be a nonempty set. A mapping d:XxX⟶ ℂ is called a complex valued metric on X if the following conditions are satisfied:

(CM1) $0\precsim d(x,y)$ for all $x,y\in X$ and $d(x,y)=0\Leftrightarrow x=y$.

(CM2) $d(x,y)=d(y,x)$ for all $x,y\in X$

(CM3) $d(x,y)\precsim d(x,z)+d(z,y)$ for all $x,y,z\in X$.

In this case,we say that (X,d) is a complex valued metric space.

It is obvious that this concept is generalization of the classic metric. In fact, if d:XxX⟶ ℝ satisfies((CM1)-(CM3)), then this d is a metric in the classical sense, that is, the following conditions are satisfies:

(M1) $0\leq d(x,y)$ for all $x,y\in X$ and $d(x,y)=0\Leftrightarrow x=y$.

(M2) $d(x,y)=d(y,x)$ for all $x,y\in X$

(M3) $d(x,y)\leq d(x,z)+d(z,y)$ for all $x,y,z\in X$.

There are so many more different and interesting type of metric spaces and classical theories of metric space for example see[3,4].

Definition 2 Let ℂ be a complex valued metric space,

- We say that a sequence $\{x_n\}$ is said to be a Cauchy sequence be a sequence in x ∈X If for every $\varepsilon\in$ ℂ, with $0\prec\varepsilon$ there is $n_0\in$ ℕsuch that for all $n>n_0$ such thatd$(x_n,x_m)\prec\varepsilon$.
- We say that a sequence $\{x_n\}$ converges to an element xIf for every x∈ ℂ, with $0\prec\varepsilon$there exist an integer $n_0\in$ ℕsuch that for all $n>n_0$ such that $d(x_n,x)\prec\varepsilon$ and we write $x_n \underset{d}{\rightarrow} x$.
- We say that (x,d) is complete if every Cauchy sequence in X converges to a point in X.

The following fact is summarized from Azam,fisher and Khan"s paper[2]. In fact,(b and c of preposition 1.3 are their lemmas 2 and 3.

II. MAIN RESULT

Let (X,d) be a complex valued metric space.
Let family of nonempty,closed and bounded subsets of a complex valued metric space is denoted by CB(X).

From now on,we denote s(z_1)={z_2∈ ℂ: z_1≾ z_2} for z1∈ ℂ, and s(a,B)=$\bigcup_{b\in B} s(d(a,b)) = \bigcup_{b\in B}\{z \in ℂ: d(a,b) \precsim z\}$for a ∈ X and B ∈ CB(X).

For A,B∈ CB(X),we denote

s(a,b)=$(\bigcup_{a\in A} s(a,B) \cap (\bigcup_{b\in B} s(b,A)$.

Theorem : let (X,d) be a complete complex valued metric space and let S,T:X→CB(X) e multivalued mapping with greatest lower bound property such that

(1) $$\mathrm{Ad}(x,y)+\mathrm{Bd}(x,Ty)+\mathrm{Cd}(y,Sx)+\frac{\mathrm{Dd}(x,Ty)d(y,Sx)}{1+d(x,y)}+\frac{\mathrm{Ed}(x,Sx)dx,Ty)}{1+d(x,y)}+\frac{\mathrm{Fd}(y,Sx)d(y,Ty)}{1+d(x,y)} \in s(Sx,Ty)$$ ∀x, y ∈ X

also,

(2) $$A+B+C+2D+2E+2F<1.$$

Then S and T have a comman fixed point also A, B, C, D, E, F are nonnegative with the (1)property.

Proof:let x_0∈ X and x_1∈Sx_0, from(1)

$$\mathrm{Ad}(x_0,x_1)+\mathrm{Bd}(x_0,Tx_1)+\mathrm{Cd}(x_1,Sx_0)+\frac{\mathrm{Dd}(x_0,Tx_1)d(x_1,Sx_1)}{1+d(x_0,x_1)}+\frac{\mathrm{Ed}(x_0,Sx_0)d(x_0,Tx_1)}{1+d(x_0,x_1)}+\frac{\mathrm{Fd}(x_1,Sx_0)d(x_1,Tx1)}{1+d(x_0,x_1)} \in$$ s(Sx_0,Tx_1) ∀x, y ∈ X

This implies that,

$$\mathrm{Ad}(x_0,x_1)+\mathrm{Bd}(x_0,Tx_1)+\mathrm{Cd}(x_1,Sx_0)+\frac{\mathrm{Dd}(x_0,Tx_1)d(x_1,Sx_1)}{1+d(x_0,x1)}+\frac{\mathrm{Ed}(x_0,Sx_0)dx_0,Tx_1)}{1+d(x_0,x_1)}+\frac{\mathrm{Fd}(y,Sx)d(y,Ty)}{1+d(x_0,x_1)} \in \left(\bigcap_{x\in Sx_0} s(x,Tx_1)\right)$$

$$\mathrm{Ad}(x_0,x_1)+\mathrm{Bd}(x_0,Tx_1)+\mathrm{Cd}(x_1,Sx_0)+\frac{\mathrm{Dd}(x_0,Tx_1)d(x_1,Sx_1)}{1+d(x_0,x_1)}+\frac{\mathrm{Ed}(x_0,Sx_0)dx_0,Tx_1)}{1+d(x_0,x_1)}+\frac{\mathrm{Fd}(y,Sx)d(y,Ty)}{1+d(x_0,x_1)} \in s(x,Tx_1)\ \ \forall x \in Sx_0$$

Since x_1∈Sx_0, such that

$$\mathrm{Ad}(x_0,x_1)+\mathrm{Bd}(x_0,Tx_1)+\mathrm{Cd}(x_1,Sx_0)+\frac{\mathrm{Dd}(x_0,Tx_1)d(x_1,Sx_1)}{1+d(x_0,x_1)}+\frac{\mathrm{Ed}(x_0,Sx_0)dx_0,Tx_1)}{1+d(x_0,x_1)}+\frac{\mathrm{Fd}(y,Sx)d(y,Ty)}{1+d(x_0,x_1)} \in s(x,Tx_1)$$

And

$$\mathrm{Ad}(x_0,x_1)+\mathrm{Bd}(x_0,Tx_1)+\mathrm{Cd}(x_1,Sx_0)+\frac{\mathrm{Dd}(x_0,Tx_1)d(x_1,Sx_1)}{1+d(x_0,x_1)}+\frac{\mathrm{Ed}(x_0,Sx_0)dx_0,Tx_1)}{1+d(x_0,x_1)}+\frac{\mathrm{Fd}(y,Sx)d(y,Ty)}{1+d(x_0,x_1)} \in s(x,Tx_1) = \bigcup_{x\in Tx_1} s(d(x_1,x))$$

So there exist some x_2∈ Tx_1

such that

$$\mathrm{Ad}(x_0,x_1)+\mathrm{Bd}(x_0,Tx_1)+\mathrm{Cd}(x_1,Sx_0)+\frac{\mathrm{Dd}(x_0,Tx_1)d(x_1,Sx_1)}{1+d(x_0,x1)}+\frac{\mathrm{Ed}(x_0,Sx_0)dx_0,Tx_1)}{1+d(x_0,x_1)}+\frac{\mathrm{Fd}(y,Sx)d(y,Ty)}{1+d(x_0,x_1)} \in s(x_1,x_2)$$

That is,

$$d(x_1,x_2) \precsim \mathrm{Ad}(x_0,x_1)+\mathrm{Bd}(x_0,Tx_1)+\mathrm{Cd}(x_1,Sx_0)+\frac{\mathrm{Dd}(x_0,Tx_1)d(x_1,Sx_1)}{1+d(x_0,x_1)}+\frac{\mathrm{Ed}(x_0,Sx_0)d(x_0,Tx_1)}{1+d(x_0,x_1)}+\frac{\mathrm{Fd}(x_1,Sx_0)d(x_1,Tx_1)}{1+d(x_0,x_1)}$$

Since,S(X)⊆X and T(X)⊆X we can construct a sequence {x_k} in X such that,

x_1=Sx_0 and x_2=Tx_1 also in general

(3) x_{2k+1}=Sx_{2k} and x_{2k+2}=Tx_{2k+1} ∀ k≥0

From the definition of {x_k} and (1) we can construct

d(x_{2k+1}, x_{2k+2})=d(Sx_{2k},Tx_{2k+1})

$$\precsim \mathrm{Ad}(x_{2k},x_{2k+1})+\mathrm{Bd}(x2k,Tx_{2k+1})+\mathrm{Cd}(x_{2k+1},Sx_{2k})+\frac{\mathrm{Dd}(x_{2k},Tx_{2k+1})d(x_{2k+1}Abbas,,Tx_{2k+1})}{1+d(x_{2k},x_{2k+1})}+\frac{\mathrm{Ed}(x_{2k},x_{2k})d(x_{2k},Tx_{2k+1})}{1+d(x_{2k},x_{2k+1})}+\frac{\mathrm{Fd}(x_{2k+1},Sx_{2k})d(x_{2k+1},Tx_{2k+1})}{1+d(x_{2k},x_{2k+1})}$$

Since,

x_{2k+1}=Sx_{2k}

Implies,

d(x_{2k+1},Sx_{2k})=0

implies,

$$d(Sx_{2k},Tx_{2k+1}) \precsim \mathrm{Ad}(x_{2k},x_{2k+1})+\mathrm{Bd}(x_{2k},Tx_{2k+1})+\frac{\mathrm{Dd}(x_{2k},Tx_{2k+1})d(x_{2k+1},Tx_{2k+1})}{1+d(x_{2k},x_{2k+1})}+\frac{\mathrm{Ed}(x_{2k},Sx_{2k})d(x_{2k},Tx_{2k+1})}{1+d(x_{2k},x_{2k+1})}$$

as we know that,|1+d(x_{2k},x_{2k+1}|>|d(x_{2k},x_{2k+1})|

$|d(Sx_{2k},Tx_{2k+1})| \leq A|d(x_{2k},x_{2k+1})| + B|d(x_{2k},Tx_{2k+1})| + \frac{D|d(x_{2k}Tx_{2k+1})||d(x_{2k+1},Tx_{2k+1})|}{|1+d(x_{2k},x_{2k+1})|} + \frac{E|d(x_{2k},Sx_{2k})||d(x_{2k},Tx_{2k+1})|}{|1+d(x_{2k},x_{2k+1})|}$

$= A|d(x_{2k},x_{2k+1})| + B|d(x_{2k},x_{2k+2})| + + \frac{D|d(x_{2k},x_{2k+2})||d(x_{2k}+1,x_{2k+2})|}{|1+d(x_{2k},x_{2k+1})|} + \frac{E|d(x_{2k},x_{2k+1})||x_{2k},x_{2k+2})|}{|1+d(x_{2k},x_{2k+1})|}$

$< A|d(x_{2k},x_{2k+1})| + B|d(x_{2k},x_{2k+2})| + \frac{D|d(x_{2k},x_{2k+2})||d(x_{2k},x_{2k+2})|}{|1+d(x_{2k},x_{2k+2})|} + E|d(x_{2k},x_{2k+2})|$

Since,

$d(x_{2k},x_{2k+2}) \leq d(x_{2k},x_{2k+1}) + d(x_{2k+1},x_{2k+2})$

$< A|d(x_{2k},x_{2k+1})| + B|d(x_{2k},x_{2k+2})| + \frac{D[|d(x_{2k},x_{2k+1})|+|d(x_{2k+1},x_{2k+2})|]|d(x_{2k+1},x_{2k+2})|}{|1+d(x_{2k},x_{2k+1})|}$

$< A|d(x_{2k},x_{2k+1})| + B|d(x_{2k},x_{2k+2})| + D[|d(x_{2k+1},x_{2k+2})| + \frac{|d(x_{2k+1},x_{2k+2}|^2}{|1+d(x_{2k},x_{2k+1})|}] + E|d(x_{2k},x_{2k+2})|$

It follows that,

$< A|d(x_{2k},x_{2k+1})| + B|d(x_{2k},x_{2k+2})| + 2D|d(x_{2k+1},x_{2k+2}| + E|d(x_{2k},x_{2k+2})|$

$|d(x_{2k+1},x_{2k+2})| - 2D|d(x_{2k+1},x_{2k+2}| < A|d(x_{2k},x_{2k+1})| + B|d(x_{2k},x_{2k+2})| + E|d(x_{2k},x_{2k+2})|$

$|d(x_{2k+1},x_{2k+2})|-[1-2D] < A|d(x_{2k},x_{2k+1})| + |d(x_{2k},x_{2k+1})|(B+E) + |d(x_{2k+1},x_{2k+2})|(B+E)$

$|d(x_{2k+1},x_{2k+2})|[1-2d-B-E] < (A+B+E)||d(x_{2k},x_{2k+1})|$

$|d(x_{2k+1},x_{2k+2})| < \frac{(A+B+E)}{[1-2d-B-E]} |d(x_{2k},x_{2k+1})|$

Similarly;

$d(x_{2k+2}, x_{2k+3}) = d(x_{2k+3},x_{2k+2}) = d(Sx_{2k+2},Tx_{2k+1})$

$\precsim Ad(x_{2k+2},x_{2k+1}) + Bd(x_{2k+2},Tx_{2k+1}) + Cd(x_{2k+1},Sx_{2k+2}) + \frac{Dd(x2k+2,Tx2k+2)d(x_{2k+1},Tx_{2k+1})}{1+d(x_{2k+2},x_{2k+1})} + \frac{Ed(x_{2k+2},Sx_{2k+2})d(x_{2k+2},Tx_{2k+1})}{1+d(x_{2k+2},x_{2k+1})} + \frac{Fd(x_{2k+1},Sx_{2k+2})d(x_{2k+1},Tx_{2k+1})}{1+d(Sx_{2k+2},x_{2k+1})}$

Since,

$x_{2k+2}=Tx_{2k+1}$ implies that $d(x_{2k+2},Tx_{2k+1})=0$

therefore,

$d(x_{2k+2},x_{2k+3}) \leq Ad(x_{2k+2},x_{2k+1}) + Cd(x_{2k+1},Sx_{2k+2}) + \frac{Dd(x_{2k+2},Tx_{2k+1})d(x_{2k+1},x_{2k+2})}{1+d(x_{2k+2},x_{2k+1})} + \frac{Fd(x_{2k+1},x_{2k+2})d(x2k+1,Tx_{2k+1})}{1+d(x_{2k+2},x_{2k+1})}$

$|1+d(x_{2k+2},x_{2k+1}| > |d(x_{2k+2},x_{2k+1})|$

We have,

$d(x_{2k+2},x_{2k+3}) \leq A|d(x_{2k+2},x_{2k+1})| + C|d(x_{2k+1},Sx_{2k+2})| + \frac{D|d(x_{2k+2},Tx_{2k+1})||d(x_{2k+1},x_{2k+2})|}{|1+d(x_{2k+2},x_{2k+1})|} + \frac{F|d(x_{2k+1},x_{2k+2})||d(x2k+1,Tx_{2k+1})|}{|1+d(x_{2k+2},x_{2k+1})|}$

usimg (3)

$= A|d(x_{2k+2},x_{2k+1})| + C|d(x_{2k+1},Sx_{2k+3})| + \frac{D|d(x_{2k+2},x_{2k+1})||dd(x_{2k+2},x_{2k+2})|}{|1+d(x_{2k+2},x_{2k+1})|} + \frac{F|d(x_{2k+3},x_{2k+1})||d(x_{2k+2},x_{2k+1})|}{|1+d(x_{2k+2},x_{2k+1})|}$

$< A|d(x_{2k+2},x_{2k+1})| + C|d(x_{2k+1},Sx_{2k+3})| + F|d(x_{2k+3},x_{2k+1})|$

Using triangle inequality,

$< A|d(x_{2k+2},x_{2k+1})| + C|d(x_{2k+1},Sx_{2k+2})| + C|d(x_{2k+2},Sx_{2k+3})| + F|d(x_{2k+2},x_{2k+1})| + F|d(x_{2k+2},x_{2k+3})|$

$|d(x_{2k+2},x_{2k+3})| - C|d(x_{2k+2},Sx_{2k+3})| - F|d(x_{2k+2},x_{2k+3})| + \leq A|d(x_{2k+2},x_{2k+1})| + C|d(x_{2k+1},Sx_{2k+2})| + F|d(x_{2k+1},x_{2k+2})|$

$|d(x_{2k+2},x_{2k+3})|[1-C-F] < (A+C+F)|d(x_{2k+2},x_{2k+1})|$

$|d(x_{2k+1},x_{2k+2})| < \frac{(A+C+F)}{[1-C-F]} |d(x_{2k+2},x_{2k+1})|$

Putting $k = \max\left\{\frac{(A+C+F)}{[1-C-F]}, \frac{(A+B+E)}{[1-2d-B-E]}\right\}$

We obtain,

$d(x_n,x_{n+1}) \leq k^1|d(x_{n-1},x_n)|$

$\leq k^2|d(x_{n-2},x_{n-1})|$

$\leq k^3|d(x_{n-3},x_{n-2})|$

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$\leq k^n|d(x_0,x_1)|\forall\ n$

Thus for any $n\in\mathbb{N}$ we have

$|d(x_n,x_m)| \leq|d(x_n,x_{n+1})| \leq|d(x_n,x_{n+1})| \leq|d(x_{n+1},x_{n+2})|\ldots\ldots\ldots \leq|d(x_{n-1},x_m)|$

$\leq (k^{n+1}+k^{n+2}+k^{n+3}+k^{n+4}+\ldots.k^{m-1})|d(x_0,x_1)|$

$\leq \left(\frac{k^{\wedge}n}{1-k}\right)|d(x_0,x_1)|$

And so,

$|d(x_n,x_m)| \leq \left(\frac{k^{\wedge}n}{1-k}\right)|d(x_0,x_1)|\to 0$

As $m,n\to\infty$

This implies that$\{x_n\}$ is a Cauchy sequence in X. since X is complete, there exist $v\in X$ such that $x_n\to v$ as $n\to\infty$, we now show that $v\in Tv$ and $v\in Sv$ we get,

$\mathrm{Ad}(x_{2n},\mathrm{v}) + \mathrm{Bd}(x_{2n},\mathrm{Tv}) + \mathrm{Cd}(\mathrm{v},\mathrm{S}x_{2n}) + \frac{\mathrm{Dd}(x_{2n},\mathrm{Tv})\mathrm{d}(\mathrm{v},\mathrm{S}x_{2n})}{1+\mathrm{d}(x_{2n},\mathrm{v})}+\frac{\mathrm{Ed}(\mathrm{v},\mathrm{S}x_{2n})\mathrm{d}(x_{2n},\mathrm{Tv})}{1+\mathrm{d}(x_{2n},\mathrm{v})}+\frac{\mathrm{Fd}(\mathrm{v},\mathrm{S}x_{2n})\mathrm{d}(\mathrm{v},\mathrm{Tv})}{1+\mathrm{d}(x_{2n},\mathrm{v})} \in s(Sx_{2n},Tv)\ \forall x,y\in X$

This implies that,

$\mathrm{Ad}(x_{2n},\mathrm{v}) + \mathrm{Bd}(x_{2n},\mathrm{Tv}) + \mathrm{Cd}(\mathrm{v},\mathrm{S}x_{2n}) + \frac{\mathrm{Dd}(x_{2n},\mathrm{Tv})\mathrm{d}(\mathrm{v},\mathrm{S}x_{2n})}{1+\mathrm{d}(x_{2n},\mathrm{v})}+\frac{\mathrm{Ed}(\mathrm{v},\mathrm{S}x_{2n})\mathrm{d}(x_{2n},\mathrm{Tv})}{1+\mathrm{d}(x_{2n},\mathrm{v})}+\frac{\mathrm{Fd}(\mathrm{v},\mathrm{S}x_{2n})\mathrm{d}(\mathrm{v},\mathrm{Tv})}{1+\mathrm{d}(x_{2n},\mathrm{v})} \in (\cap_{x\in Sx_{2n}} s(x,Tv))$

And we have

$\mathrm{Ad}(x_{2n},\mathrm{v}) + \mathrm{Bd}(x_{2n},\mathrm{Tv}) + \mathrm{Cd}(\mathrm{v},\mathrm{S}x_{2n}) + \frac{\mathrm{Dd}(x_{2n},\mathrm{Tv})\mathrm{d}(\mathrm{v},\mathrm{S}x_{2n})}{1+\mathrm{d}(x_{2n},\mathrm{v})}+\frac{\mathrm{Ed}(\mathrm{v},\mathrm{S}x_{2n})\mathrm{d}(x_{2n},\mathrm{Tv})}{1+\mathrm{d}(x_{2n},\mathrm{v})}+\frac{\mathrm{Fd}(\mathrm{v},\mathrm{S}x_{2n})\mathrm{d}(\mathrm{v},\mathrm{Tv})}{1+\mathrm{d}(x_{2n},\mathrm{v})} \in s(x,Tv)\forall\ x\in Sx_{2n}$

Since $x_{2n+1}\in Sx_{2n}$, so we have,

$\mathrm{Ad}(x_{2n},\mathrm{v}) + \mathrm{Bd}(x_{2n},\mathrm{Tv}) + \mathrm{Cd}(\mathrm{v},\mathrm{S}x_{2n}) + \frac{\mathrm{Dd}(x_{2n},\mathrm{Tv})\mathrm{d}(\mathrm{v},\mathrm{S}x_{2n})}{1+\mathrm{d}(x_{2n},\mathrm{v})}+\frac{\mathrm{Ed}(\mathrm{v},\mathrm{S}x_{2n})\mathrm{d}(x_{2n},\mathrm{Tv})}{1+\mathrm{d}(x_{2n},\mathrm{v})}+\frac{\mathrm{Fd}(\mathrm{v},\mathrm{S}x_{2n})\mathrm{d}(\mathrm{v},\mathrm{Tv})}{1+\mathrm{d}(x_{2n},\mathrm{v})} \in s(x_{2n+1},Tv)$

By definition we obtain,

$\mathrm{Ad}(x_{2n},\mathrm{v}) + \mathrm{Bd}(x_{2n},\mathrm{Tv}) + \mathrm{Cd}(\mathrm{v},\mathrm{S}x_{2n}) + \frac{\mathrm{Dd}(x_{2n},\mathrm{Tv})\mathrm{d}(\mathrm{v},\mathrm{S}x_{2n})}{1+\mathrm{d}(x_{2n},\mathrm{v})}+\frac{\mathrm{Ed}(\mathrm{v},\mathrm{S}x_{2n})\mathrm{d}(x_{2n},\mathrm{Tv})}{1+\mathrm{d}(x_{2n},\mathrm{v})}+$

$\frac{\mathrm{Fd}(\mathrm{v},\mathrm{S}x_{2n})\mathrm{d}(\mathrm{v},\mathrm{Tv})}{1+\mathrm{d}(x_{2n},\mathrm{v})} \in \mathrm{s(x2n+1,Tv)}= \left(\cup_{p\in Tv} s(d(x2n+1,p))\right)$

$\exists$some $v_n\in Tv$ such that

$\mathrm{Ad}(x_{2n},\mathrm{v}) + \mathrm{Bd}(x_{2n},\mathrm{Tv}) + \mathrm{Cd}(\mathrm{v},\mathrm{S}x_{2n}) + \frac{\mathrm{Dd}(x_{2n},\mathrm{Tv})\mathrm{d}(\mathrm{v},\mathrm{S}x_{2n})}{1+\mathrm{d}(x_{2n},\mathrm{v})}+\frac{\mathrm{Ed}(\mathrm{v},\mathrm{S}x_{2n})\mathrm{d}(x_{2n},\mathrm{Tv})}{1+\mathrm{d}(x_{2n},\mathrm{v})}+$

$$\frac{\mathrm{Fd}(\mathrm{v},\mathrm{S}x_{2n})\mathrm{d}(\mathrm{v},\mathrm{Tv})}{1+\mathrm{d}(x_{2n},\mathrm{v})} \in \mathrm{s(x2n+1,Tv)}$$

$$\in s\big(d(x2n+1,vn)\big)$$

i.e.

$d(x_{2n+1},v_n)\precsim \mathrm{Ad}(x_{2n},\mathrm{v}) + \mathrm{Bd}(x_{2n},\mathrm{Tv}) + \mathrm{Cd}(\mathrm{v},\mathrm{S}x_{2n}) + \frac{\mathrm{Dd}(x_{2n},\mathrm{Tv})\mathrm{d}(\mathrm{v},\mathrm{S}x_{2n})}{1+\mathrm{d}(x_{2n},\mathrm{v})}+\frac{\mathrm{Ed}(\mathrm{v},\mathrm{S}x_{2n})\mathrm{d}(x_{2n},\mathrm{Tv})}{1+\mathrm{d}(x_{2n},\mathrm{v})}+\frac{\mathrm{Fd}(\mathrm{v},\mathrm{S}x_{2n})\mathrm{d}(\mathrm{v},\mathrm{Tv})}{1+\mathrm{d}(x_{2n},\mathrm{v})}$By using the greatest lower bound property of S and T we have,

$d(x_{2n+1},v_n)\leq \mathrm{Ad}(x_{2n},x_{2n+1}) + \mathrm{Bd}(x_{2n},v_n) + \mathrm{Cd}(\mathrm{v},x_{2n+1}) + \frac{\mathrm{Dd}(x_{2n},v_n)\mathrm{d}(\mathrm{v},x_{2n+1})}{1+\mathrm{d}(x_{2n},\mathrm{v})}+\frac{\mathrm{Ed}(\mathrm{v},x_{2n+1})\mathrm{d}(x_{2n},v_n)}{1+\mathrm{d}(x_{2n},\mathrm{v})}+$

$$\frac{\mathrm{Fd}(\mathrm{v},x_{2n+1})\mathrm{d}(\mathrm{v},v_n)}{1+\mathrm{d}(x_{2n},\mathrm{v})}$$

Now using triangle inequality;

$d(x_{2n+1},v_n)\leq\mathrm{Ad}(x_{2n},\mathrm{v}) + \mathrm{Bd}(x_{2n},v_n) + \mathrm{Bd}(x_{2n},v_n) + \mathrm{Cd}(\mathrm{v},x_{2n+1}) + \frac{\mathrm{Dd}(x_{2n},v_n)\mathrm{d}(\mathrm{v},x_{2n+1})}{1+\mathrm{d}(x_{2n},\mathrm{v})}+\frac{\mathrm{Ed}(\mathrm{v},x_{2n+1})\mathrm{d}(x_{2n},v_n)}{1+\mathrm{d}(x_{2n},\mathrm{v})}+\frac{\mathrm{Fd}(\mathrm{v},x_{2n+1})\mathrm{d}(\mathrm{v},v_n)}{1+\mathrm{d}(x_{2n},\mathrm{v})}$

$d(x_{2n+1},v_n)\leq\left(\frac{B}{1-B}\right)d(x_{2n},x_{2n+1})+\left(\frac{A}{1-B}\right)d(x_{2n},v)+\left(\frac{C}{1-B}\right)\ \mathrm{d}(\mathrm{v},x_{2n+1}) + \left(\frac{D}{1-B}\right)\frac{\mathrm{d}(x_{2n},v_n)\mathrm{d}(\mathrm{v},x_{2n+1})}{1+\mathrm{d}(x_{2n},\mathrm{v})}+\left(\frac{E}{1-B}\right)\frac{\mathrm{d}(\mathrm{v},x_{2n+1})\mathrm{d}(x_{2n},v_n)}{1+\mathrm{d}(x_{2n},\mathrm{v})}+\left(\frac{F}{1-B}\right)\frac{\mathrm{d}(\mathrm{v},x_{2n+1})\mathrm{d}(\mathrm{v},v_n)}{1+\mathrm{d}(x_{2n},\mathrm{v})}$

again using triangular inequality,

$d(v,v_n)\leq d(v,x_{2n+1})+d(x_{2n+1},v_n)$

$\leq d(v,x_{2n+1})+\left(\frac{B}{1-B}\right)d(x_{2n},x_{2n+1})+\left(\frac{A}{1-B}\right)d(x_{2n},v)+\left(\frac{C}{1-B}\right)\ \mathrm{d}(\mathrm{v},x_{2n+1}) + \left(\frac{D}{1-B}\right)\frac{\mathrm{d}(x_{2n},v_n)\mathrm{d}(\mathrm{v},x_{2n+1})}{1+\mathrm{d}(x_{2n},\mathrm{v})}+\left(\frac{E}{1-B}\right)\frac{\mathrm{d}(\mathrm{v},x_{2n+1})\mathrm{d}(x_{2n},v_n)}{1+\mathrm{d}(x_{2n},\mathrm{v})}+\left(\frac{F}{1-B}\right)\frac{\mathrm{d}(\mathrm{v},x_{2n+1})\mathrm{d}(\mathrm{v},v_n)}{1+\mathrm{d}(x_{2n},\mathrm{v})}$

It follows that,

$|d(v,v_n)|\leq|d(v,x_{2n+1})|+\left|\left(\frac{B}{1-B}\right)\mathrm{d}(x_{2n},x_{2n+1}) + \left(\frac{A}{1-B}\right)\mathrm{d}x_{2n},\mathrm{v}) + \left(\frac{C}{1-B}\right)\ \mathrm{d}(\mathrm{v},x_{2n+1}) +\right.$

$$\left(\frac{D}{1-B}\right)\frac{d(x_{2n},v_n)d(v,x_{2n+1})}{1+d(x_{2n},v)} + \left(\frac{E}{1-B}\right)\frac{d(v,x_{2n+1})d(x_{2n},v_n)}{1+d(x_{2n},v)} + \left(\frac{F}{1-B}\right)\frac{d(v,x_{2n+1})d(v,v_n)}{1+d(x_{2n},v)}\Bigg|$$

By letting $n\to\infty$,we have $|d(v,v_n)|\to 0$ as $n\to\infty$.

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