Reliability Analysis of Pier Scour at Gravel-Bed Rivers Using FORM

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Abstract- **A correct and reliable assessment of scour depth at bridge pier is of prime importance in safe and economical design of bridge pier foundation. An attempt was made to develop a reliability-based scour depth prediction model at bridge pier in gravel bed-rivers. A developed deterministic scour predictions model by Melville and Sutherland (1988) and an object oriented constrained optimization using spread sheet algorithm for FORM have been used for the reliability analysis in present study. To achieve a desired safety level in the design of pier foundation, reliability based safety factor is proposed. It was found that the reliability index increases rapidly with the increase in safety factor whereas, an increase in safety factor results in decrease in failure probability at faster rate.**

***Keywords*—Reliability, First order reliability method, Bridge Pier, Gravel bed-rivers.**

I. INTRODUCTION

Scour at the bridge foundation is the most common cause of bridge failure in the world. Bridge pier scouring is, therefore, an important issue in the safety evaluation of bridges. To avoid such failures, the pier foundation has to be deeper than the maximum possible scour depth. Hence a reliable estimate of maximum possible scour depth around a bridge pier is essential to assure a safe and economic bridge foundation design. The process of scour around bridge pier is extremely complex because it involves three- dimensional flow with the sediment transport. The boundary layer flow past a bridge element undergoes a three-dimensional separation. This separated shear layers rolls up along the obstruction to form a vortex system in front of the obstructing element, which is swept downstream by the flow. When viewed from the top, this vortex system has the characteristic shape of a horseshoe and is thus called a horseshoe vortex. The formation of a horseshoe vortex and the associated down flow around the bridge element results in increased shear stress and hence a local increase in sediment transport capacity of the flow. This leads to the development of a deep scour hole around the bridge element. The estimation of the scour extent and is depth at bridge sites therefore continue to be a major concern of the bridge designers. The process of bridge pier scour has been investigated extensively and many relationships are now available for estimation of the design scour depth for bridge piers in alluvial streams. The study on bridge pier scour in gravel bed-rivers is relatively scanty.

II. CHARCTERSTICS OF GRAVEL BED-RIVERS

The bed material of gravel bed-rivers is usually characterized by relatively large median size and large geometric standard deviation. During relatively large flood, all the finer particles in the bed material of such rivers move. When the discharge reduces, the coarser particles, which cannot be moved, accumulate on the bed surface and form a layer of non- movable particles on the bed. This is known as armour layer or paving. For lower discharges there is no sediment transport, since original bed material is overlain by the armour layer. The bed material can then be termed as layered or stratified material. The standard deviation of the top layer is much smaller than that of the original bed material and the sediment is usually coarser in size, and the top layer that has a thickness of one or two times the largest size in the bed material. In case of bridge pier founded in gravel-bed rivers, as the scour progresses during the passage of flood, coarser particles will accumulate in the scour hole than that in alluvial rivers which is generally have relatively finer bed material size and more uniform gradation [1].

In the upper reaches, riverbeds are commonly composed of a mixture of different sizes of sand and gravel. Under the varied stream flow velocities, a process of armoring on the riverbeds commences, resulting in an exposure of coarser particles due to washing out if the finer fraction. The armour layer is of concerns to estimate scour depth at bridge pier where the pier is embedded in the sand bed overlain by layer of gravels. A larger scour depth develops at pier embedded in an armored bed unless a secondary armor layer developed within the scour hole. This scouring potential was first recognized by [2] and the scour at circular piers in armored beds was studied by [3].

The codes namely [4] and [5] that are to be followed in engineering practice in India for design against scour, recommended the use Lacey's formula for computation of the regime depth and involving discharge intensity q, with a silt factor of 24 for scour computation in gravel bed-rivers. The scour depth below the high flood level (HFL) is estimated to be equal to twice the Lacey's depth of flow in the river. This raises the basic equation as to whether gravel-bed river data follow Lacey's relation for depth.

Analytical, semi-empirical or empirical scour equations based on the mechanics of scour, the dimensional analysis and the data correlation of laboratory experiments or field observations were developed [6], [7], [8], [9], and [10]. Scour Phenomena are complex in nature and consequently experimental investigation were limited to certain aspects by overlooking other aspects. As a result a simplified experimentation may misinterpret the prototype conditions. The calibration of scour prediction models with the field data is restricted mainly due to lack of relevant size and the precision of the field data [11]. The existing scour equations have a considerable uncertainty due to uncertain flow, sediment and structural parameters, therefore, the inherent uncertainties in various parameters suggest that pier safety should be ensured in a probabilistic sense. A reliability analysis may provide a quantitative estimation of the pier safety against scouring.

The probabilistic approach to the scour depth prediction at bridge pier was applied, among others, by [12], [13], [14], [15], [16] and [17]. [12] And [13] were probably the first who raise the issue of the reliability-based pier assessment presenting a risk based-method by incorporating uncertainty into the bridge pier design.

III. SCOUR DEPTH PREDICTOR IN GRAVEL BED-

There are no set procedures available in literature for estimation of scour depth in gravel bed-rivers. This is primarily due to nonavailably of scour data for prototype structures. The following are the few methods for estimating Bridge Pier Scour in gravel bed-rivers,

- Raudkivi (1986)
- Kothyari et.al (1992)
- Kothyari New 'f' (silt factor)
- IRC-78 (2000)
- Melville and Sutherland (1988)
- Sheppard et.al (2004)
- Lanca et.al (2013).

In the present study for the computation of the deterministic scour depth at bridge piers, Melville and Sutherland (1988) approach has been used. [7]

IV. MELVILLE AND SUTHERLAND EQUATIONS

Melville and Sutherland (1988) have proposed a method for estimating the equilibrium scour depth at bridge piers. The method assumes that the largest possible scour depth around a bridge pier is given by

$$d_s = 2.4b \qquad (1)$$

Where d_s is the maximum scour depth below general bed level and b is pier width .This scour depth is then, reduced by multiplying factors which depend on whether it is the clear water scour, flow depth is shallow, and sediment is graded. The multiplying factors have been developed from the analysis of extensive laboratory data covering a wide range of pertinent variables.

The average critical velocity U_c is determined first by obtaining critical shear velocity u_{*c} using Shields' criteria and then using logarithmic relation between U_c/u_{*c} and y/d_{50}. Thus

$$u_{*_c} = 0.03 d_{50}^{0.50} \qquad (2).$$

Where d_{50} is the particle size for 50 percentile weights and y is the flow depth. u_{*c} is in m/s and d_{50} is in mm.

The logarithmic relation for U_c is

$$U_c/u_{*_c} = 5.75\, log\left(\frac{5.53y}{d_{50}}\right) \qquad (3)$$

Live bed scour is supposed to take place $U > U_c$. For non-uniform sediment U_c would naturally depend on d_{50} and the geometric standard deviation of bed material $\sigma_g = d_{84}/d_{50}$. However, it is assumed that up to $\sigma_g = 2.5$ using d_{50} and above expressions would give satisfactory estimate of U_c. When material is much more non-uniform, limiting armour condition is defined which gives the coarsest or most stable armour bed for given bed material and is characterized by average critical velocity U_{ca}. If $U > U_{ca}$ no armour would form; if $U < U_{ca}$, the d_{50} of armour layer designated as d_{50a} will be smaller than that formed at U_{ca}. According to [18], U_{ca} depends on d_{max} of the bed material, d_{max} being determined from the relation.

$$d_{max} = d_{50}\sigma_g^m \qquad (4)$$

Where, d_{max} is bed material particle size distribution, $\sigma_g{}^m$ is the mean standard geometric deviation of bed material. m =1.28 if $d_{max} = d_{90}$; m =1.65 if $d_{max} = d_{95}$; m =2.06 if $d_{max} = d_{98}$; m = 2.34if $d_{max} = d_{99}$.

These values are obtained by assuming the bed material size distribution to follow log-normal law. Further

$$d_{50a} = \frac{d_{max}}{1.8} \qquad (5)$$

For this size d_{50a}, critical shear velocity u_{*ca} can be determined from equation 2 and critical average velocity U_{ca} from equation 3. Further, the limiting condition for scour may be assessed with the value of U_a as given below

$$U_a = 0.80 U_{ca} \qquad (6)$$

One should check if $U_a > U_c$. If $U_a < U_c$ one should use U_c.

Melville and Sutherland (1988) proposed a method for estimation of the equilibrium scour depth at bridge pier in the following form

$$d_S/b = K_i\, K_y\, K_d K_s\, K_\theta \qquad (7)$$

Coefficient K_i

$$K_i = 2.4\left[\frac{U-(Ua-Uc)}{U_c}\right]; \qquad \text{for clear water scour} \qquad (8)$$

$$K_i = 2.4; \qquad \text{for live bed scour} \qquad (9)$$

$$If\ \frac{U-(U_a-U_c)}{U_c} < 1.0 \quad \text{It indicates clear water scour}$$

$$If\ \frac{U-(U_a-U_c)}{U_c} > 1.0 \quad \text{It indicates live bed scour}$$

Coefficient K_y

$$K_y = 1.0 \quad , if\ \frac{y}{b} > 2.60 \qquad (10)$$

$$K_y = 0.78\left(\frac{y}{b}\right)^{0.255}, if\ \frac{y}{b} < 2.60 \quad (11)$$

Coefficient K_d

$$K_d = 1.0 \quad , if\ \frac{b}{d} > 25 \quad (12)$$

$$K_d = 0.57 log\left(2.24\frac{b}{d}\right), if\ \frac{b}{d} < 25 \quad (13)$$

V. ANALYSIS, RESULTS AND DISCUSSIONS

A study on reliability based scour depth prediction at bridge piers in grave bed-rivers has been carried out. A scour prediction equations proposed by [7] have been adopted in the present study. An object oriented constrained optimization using spreadsheet algorithm for FORM has been used for the reliability analysis. Some of the important results are analyzed, discussed and presented.

Table 1 Details of Data of river Ganga at Hridwar and scour depth based on the deterministic approach

<table>
<tr><th>Details of Data</th><th>Remarks</th></tr>
<tr><td>Average velocity, U=2.5 m/s ;
average depth of flow, y =2.8m;
Diameter of circular pier, b=2.5;
Some of the important results of the size distribution of the river bed are in the following table.

<table>
<tr><td>d(mm)</td><td>100</td><td>90</td><td>45</td><td>20</td><td>1.0</td></tr>
<tr><td>% finer</td><td>100</td><td>85</td><td>50</td><td>16</td><td>5</td></tr>
</table>
u_{*c}=0.2m/s; Based on Shields' criterion for initiation of motion;
U_C=2.8m/s; Based on Manning's-Stickler equation.</td><td>Scour depth was estimated using Melville and Sutherland (1988) approach as 4.817m</td></tr>
</table>

The gravel-bed-rivers data of river Ganga at Haridwar for the scour prediction was taken from [19] as given in Table 1.

VI. PROBABILISTIC APPROACH

The reliability assessment of any structure is concerned with the calculation and prediction of limit state violation at any stage during its entire life. In the present study limit state violation exceeding the maximum local scour depth from the depth of foundation. A limit state function is mathematically representation of a particular limit state of failure. This function assumes a negative or zero value at failure and a positive value for safety. Thus we can define the probability of limit state violation (i.e. probability of failure) as;

$$p_f = [g(x) \leq 0] \quad (14)$$

With *g(x)* is the limit state function, if the depth of foundation is d_f and maximum scour depth is d_s then performance function can be written as;

$$g(x) = d_f - d_s \quad (15)$$

Equation 15 may further be written as

$$g(x) = d_f - \lambda d_s(x) \quad (16)$$

Here x = (λ, b, σ, d_{50}, U, y); x is a vector of random variables and λ a model correction factor. The latter incorporates the model uncertainty associated with the pier scour model. It is estimated as the ratio of the observed to the predicted scour depth.

VII. APPLICATION OF THE PRESENT APPROACH

The statistical data of basic variables for reliability analysis are given in table 2. The mean values of these variables are adopted from the numerical example as provided table 1. Other statistical parameters such as coefficient of variation (COV) and distribution of random variables were assumed. All the random variables were assumed as independent. An object oriented constrained optimization using spreadsheet algorithm for FORM for these data set was implemented for the reliability analysis [20].

Table 2 Statistical data of governing parameters

S.N.	*Parameters*	*Distribution*	*Mean*	*COV*
1	Pier width (b)	Triangular	2.500	0.18
2	Average approach flow velocity (U)	Triangular	2.500	0.04
3	Water depth (y)	Triangular	2.800	0.03
4	Sediment size (d50)	Triangular	0.045	0.10
5	Sediment gradation (σ)	Triangular	2.000	0.10
6	Model factor (λ)	Triangular	0.930	0.05

The results of the reliability analysis are summarized in Table 3.

Table 3 Reliability Index (β) and failure probability (p_f) for different safety factors (SF)

SF	1.0	1.05	1.10	1.15	1.20	1.25	1.30	1.35
β	0.50	0.84	1.19	1.56	1.97	2.45	3.05	3.99
p_f	0.30	0.19	0.11	0.05	0.02	0.007	0.001	0.00003

VIII. RELATION OF RELIABILITY INDEX WITH FAILURE PROBABILITY WITH SAFETY FACTOR

Safety factors are used in engineering to protect a system against design uncertainties. The factor of safety for scour may be defined as the ratio of foundation depth to the scour depth as;

$$SF = \frac{d_f}{d_s} \quad (17)$$

The factor of safety is calculated above using the mean values of d_f and d_s, the randomness of scour parameters has not been taken into account. The factor of safety based on central value is not a direct measure of risk, therefore must be compared to direct risk measure in the term of the probability of failure to gain understanding of the relative safety provided by the factor of safety. The relations of reliability index and failure probability with safety factor have been shown in Fig. 1(a) and Fig. 1(b) respectively. The effect of factor of safety on reliability index and failure probability may be clearly observed. It may be observed that reliability index increase while probability of failure decreases with the increase in safety factor.

IX. RELIABILITY-BASED SAFETY FACTORS

Safety factors are widely used to incorporate uncertainties involved in the various stages of pier design and construction. Although these factors are qualitative measures of safety they do not directly indicate the "quantity or magnitude" of safety. In the present study, the reliability indices were obtained using a reliability analysis. The following equation of the safety factor SF in terms of the target (subscript T) reliability index was developed with a coefficient of determination $R^2 = 0.99$

$$SF = 0.91 + 0.18\beta_T + 0.017\beta_T^2 \tag{18}$$

This equation results in appropriate value of the safety factor for a desired pier reliability. The result is not recommended generally, because it is based on limited data set and assumes values of uncertainties. However this equation provides a decision basis of the appropriate safety factor.

X. CONCLUSIONS

Following main conclusions were made from present reliability analysis of bridge pier against scour.

(a) Reliability Index increases rapidly with the increase in safety factor.

(b) An increase in safety factor results in decrease in failure probability at faster rate.

(c) It is not only the mean value but also the uncertainties involves in various parameters play a significant role in determining the reliability of bridge pier.

(d) A reliability based safety factor in terms of reliability index has been developed.

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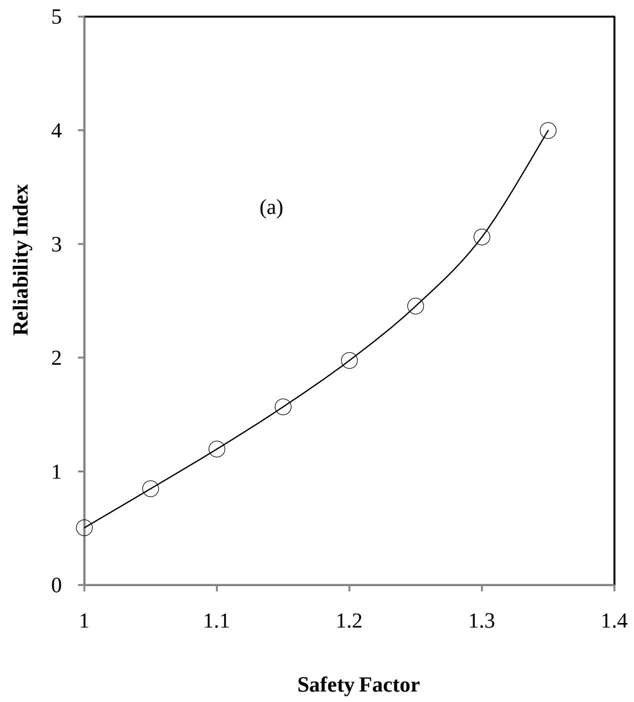

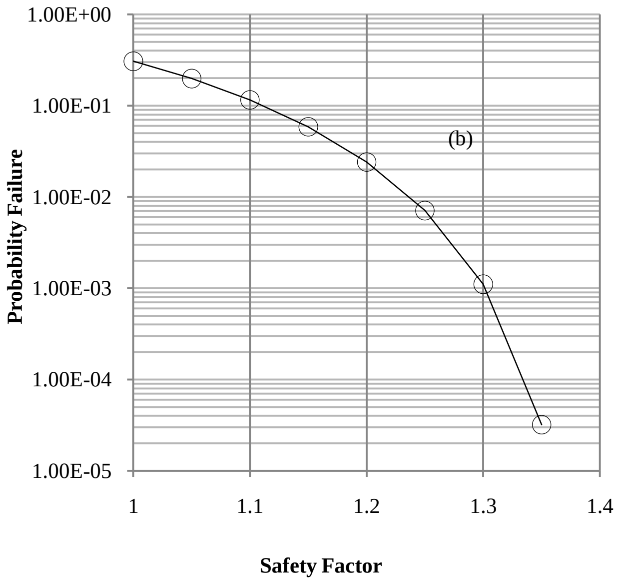

Fig. 1 (a) Relation of Reliability Index with Safety Factor (b) Relation of Probability of Failure with Safety Factor

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