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An SEIR Model for Malaria with Infective Immigrants

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Abstract. **In this paper, we develop an SEIR model for the human population and SI model for the mosquito population respectively. Sus- ceptible or uninfected humans can become infected when they are bitten by an infectious mosquito. After they are bitten, they go to the exposed class and then move on to the infectious class and then to the recov- ered class. Susceptible mosquitoes can become infected when they bite infectious humans and once they are infected they move into the infec- tious class and remain infected till death.The SEIR model proposed here takes into consideration new recruits in the population who are suscepti- ble, exposed or infective.We calculate the equilibrium points of the newly developed model and analyse the stability at these points.**

Keywords: **Susceptible, Exposed, Infective, Recovered, Infective Immigrants, Equi- librium points, Stability**

1 Introduction

Malaria is the world's most life threatening blood disease caused by the female anopheles mosquitoes. Even though efforts for eradication and control of malaria have been going on for many years, the disease keeps coming back in places which were even considered as disease-free. This is because of a number of factors and one of the main factor is human migration and travel. When people infected with malaria parasite migrate and go to areas which consist of uninfected mosquitoes, they are capable of being the source of spreading malaria in those areas. The malaria parasite is never dormant once it enters the human body, so it always causes a disease unlike some other vector-borne diseases. It stands to reason, that sick people do not usually migrate. However, since there is roughly a 10 days to 4 weeks day period between infection and the disease, there is a possibility for infected people to travel before they develop the disease and thus take the disease to a new location. As a result of this, infected immigrants have quite an impact on the spread of malaria within a population. Even if the immigrants are not bringing the disease to a new location, the number of infected mosquitoes increase because of the increase in number of infected people in that area.This is one of the reason for the number of infected persons to increase in an area.

In [**?**,**?**,**?**,**?**,**?**] various epidemiological models that include infective immigrants have been studied. Similarly, several SEIR models for vector borne diseases and malaria in particular have been developed in [**?**,**?**,**?**,**?**,**?**,**?**,**?**]. However,

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so far, no SEIR model of malaria with infective immigrants is available in the lit- erature. Hence in this paper we formulate a new SEIR model taking into account the infective immigrants.

The paper is organised as follows. In Section 2, we develop an SEIR model for the human population and an SI model for the mosquitoes. In Section 3, we calculate the equilibrium points for the model developed in Section 2. In Section 4, we study the stability of the equilibrium points.

2 Formulation of SEIR model

Let us denote the total population of human hosts as $N_h(t)$ and the total population of the female mosquitoes as $N_m(t)$. The human population $N_h(t)$ is divided into the epidemiological subclasses: Susceptible, Exposed, Infected and Recovered and they are denoted by $S_h(t)$, $E_h(t)$, $I_h(t)$ and $R_h(t)$ respectively. Thus,

$$N_h(t) = S_h(t) + E_h(t) + I_h(t) + R_h(t).$$

The mosquito population $N_m(t)$ is divided into two subclasses: Susceptible and Infected and denoted by $S_m(t)$ and $I_m(t)$ respectively. We assume that the parasite remains in the mosquito for the rest of its life and hence it remains infectious till its death. Thus,

$$N_m(t) = S_m(t) + I_m(t).$$

We now consider a model in which the new recruits that come into the population are either infective or susceptible. This new recruitment is assumed to occur through birth or immigration at constant rate λ. We further assume that a fraction φ is infective and a fraction α is exposed and the remaining fraction$(1 - \varphi - \alpha)$ is susceptible.

Using the above assumptions, we formulate the model as a system of non- linear differential equations given below:

$$\frac{dS_h}{dt} = (1 - \varphi - \alpha)\lambda - \mu_h S_h - \beta_h S_h I_m$$

$$\frac{dE_h}{dt} = \alpha\lambda + \beta_h S_h I_m - \mu_h E_h - \nu_h E_h$$

$$\frac{dI_h}{dt} = \varphi\lambda + \nu_h E_h - \gamma I_h - \alpha_h I_h - \mu_h I_h$$

$$\frac{dR_h}{dt} = \gamma I_h - \mu_h R_h$$

$$\frac{dN_h}{dt} = \lambda - \mu_h N_h - \alpha_h I_h \qquad (1)$$

$$\frac{dS_m}{dt} = \rho - \beta_m S_m I_h - \mu_m S_m - \alpha_m S_m$$

$$\frac{dI_m}{dt} = \beta_m S_m I_h - \mu_m I_m - \alpha_m I_m$$

$$\frac{dN_m}{dt} = \rho - \mu_m N_m - \alpha_m N_m$$

where

β_h=Interaction coefficient of susceptible human with infectious mosquitoes,

ν_h=Rate of progression of humans from exposed to the infectious state,

μ_h=Natural death rate of the human population,

α_h=Disease related death rate of the human population, γ=Recovery rate of human population,

β_m=Interaction coefficient of infected human with susceptible mosquitoes,

α_m=Death rate due to control measures,

μ_m=Natural death rate,

ρ=Recruitment rate of mosquitoes.

Now using $S_h + E_h + I_h + R_h = N_h$ and $S_m + I_m = N_m$ the above model reduces to

$$\frac{dE_h}{dt} = \alpha\lambda + \beta_h (N_h - E_h - I_h - R_h) I_m - \mu_h E_h - \nu_h E_h$$

$$\frac{dI_h}{dt} = \varphi\lambda + \nu_h E_h - \gamma I_h - \alpha_h I_h - \mu_h I_h$$

$$\frac{dR_h}{dt} = \gamma I_h - \mu_h R_h$$

$$\frac{dN_h}{dt} = \lambda - \mu_h N_h - \alpha_h I_h \qquad (2)$$

$$\frac{dI_m}{dt} = \beta_m (N_m - I_m) I_h - \mu_m I_m - \alpha_m I_m$$

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$$\frac{dN_m}{dt} = \rho - \mu_m N_m - \alpha_m N_m$$

The following theorem guarantees the existence and the positivity of the solution in a feasible region A.

Theorem 1. *The domain of the solution set for the system (2) is given by:*

$$A = \{(E_h, I_h, R_h, N_h, I_m, N_m) : \frac{\lambda}{\mu_h + \alpha_h} \leq N_h \leq \frac{\lambda}{\mu_h}; 0 \leq I_m \leq N_m^*\}$$

and the solutions remain non-negative in A, when

$$N_m^* = \rho - \mu_m N_m - \alpha_m N_m.$$

Proof. From fourth equation of model (2), the population $N_h(t)$ satisfies,

$$\frac{dN_h}{dt} = \lambda - \mu_h N_h - \alpha_h I_h.$$

Since $I_h(t) < N_h(t)$, we have

$$\lambda - (\mu_h + \alpha_h)N_h(t) \leq \frac{dN_h(t)}{dt} \leq \lambda - \mu_h N_h(t)$$

and hence, $\frac{\lambda}{\mu_h+\alpha_h} \leq N_h \leq \frac{\lambda}{\mu_h}$

From seventh equation of model (2) we have

$$\frac{dN_m}{dt} = \rho - \mu_m N_m - \alpha_m N_m$$

$$= \rho - (\mu_m + \alpha_m)N_m.$$

It follows that, $0 \leq \frac{dN_m}{dt} \leq \frac{\rho}{\alpha_m+\mu_m}$.

3 Equilibrium Analysis

Theorem 2. *The system (2) has three non-negative equilibria, namely,*

(a) $E_1 = (0, 0, 0, N_h^*, 0, 0)$

(b) $E_2 = (0, 0, 0, N_h^{**}, 0, N_m^{**})$

(c) $E_3 = (\hat{E}_h, \hat{I}_h, \hat{R}_h, \hat{N}_h, \hat{I}_m, \hat{N}_m)$ *provided,* $\frac{(\mu_h+\nu_h)(\gamma+\alpha_h+\mu_h)}{(\mu_h+\alpha_h)(\alpha\nu+\eta\rho)} > 0$

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Proof. (a) Existence of $E_1(0,0,0,N_h^*,0,0)$

Here N_h^* is the solution of the equation $\lambda - \mu_h N_h = 0$.

We know that $N_h^* = \frac{\lambda}{\mu_h} > 0$.

Hence equilibrium point $E_1(0,0,0,N_h^*,0,0)$ exists.

(b) Existence of $E_2(0,0,0,N_h^{**},0,N_m^{**})$

Here N_h^{**} and N_m^{**} are given by the solution of the following equations,

$$\lambda - \mu_h N_h^{**} = 0$$

and

$$\rho - \mu_m N_m^{**} - \alpha_m N_m^{**} = 0.$$

Clearly, $N_h^{**} = \frac{\lambda}{\mu_h} > 0$ and $N_m^{**} = \frac{\rho}{\mu_m + \alpha_m} > 0$.

So the equilibrium point $E_2(0,0,0,N_h^{**},0,N_m^{**})$ exists.

(c) Existence of $E_3(\hat{E}_h, \hat{I}_h, \hat{R}_h, \hat{N}_h, \hat{I}_m, \hat{N}_m)$

Let consider the following equations

$$\alpha\lambda + \beta_h(N_h - E_h - I_h - R_h)I_m - \mu_h E_h - \nu_h E_h = 0 \quad (3)$$

$$\varphi\lambda + \nu_h E_h - \gamma I_h - \alpha_h I_h - \mu_h I_h = 0 \quad (4)$$

$$\gamma I_h - \mu_h R_h = 0 \quad (5)$$

$$\lambda - \mu_h N_h - \alpha_h I_h = 0 \quad (6)$$

$$\beta_m(N_m - I_m)I_h - \mu_m I_m - \alpha_m I_m = 0 \quad (7)$$

$$\rho - \mu_m N_m - \alpha_m N_m = 0 \quad (8)$$

The non-trivial equilibrium point E_3 is got by solving the above equations;

From equation (6) we get,

$$I_h = \frac{\lambda - \mu_h N_h}{\alpha_h} \quad (9)$$

From equation (8) we get,

$$N_m = \frac{\rho}{\mu_m + \alpha_m} \quad (10)$$

From equations (5) and (9) we get,

$$R_h = \frac{\gamma(\lambda - \mu_h N_h)}{\alpha_h \mu_h} = g_h(N_h)\,say \tag{11}$$

From equations (4) and (9) we get ,

$$E_h = \frac{(\gamma + \alpha_h + \mu_h)(\lambda - \mu_h N_h) - \alpha_h \varphi \lambda}{\alpha_h \nu_h} \tag{12}$$

From equations(7) and (9) we get,

$$I_m = \frac{\beta_m N_m(\lambda - \mu_h N_h)}{\beta_m(\lambda - \mu_h N_h) + \alpha_h(\mu_m + \alpha_m)} \tag{13}$$

Now putting E_h, I_h, R_h, I_m in equation (3) we get,

$$F(N_h) = \alpha\lambda + \frac{\beta_h \beta_m N_m(\lambda - \mu_h N_h)\left[N_h - \frac{(\gamma+\alpha_h+\mu_h)(\lambda-\mu_h N_h)-\alpha_h\varphi\lambda}{\alpha_h \nu_h} - \left(\frac{\lambda-\mu_h N_h}{\alpha_h}\right) - g_h(N_h)\right]}{\beta_m(\lambda - \mu_h N_h) + \alpha_h(\mu_m + \alpha_m)} - \frac{(\mu_h + \nu_h)(\gamma + \alpha_h + \mu_h)(\lambda - \mu_h N_h) - \alpha_h \varphi \lambda}{\alpha_h \nu_h} \tag{14}$$

It is clear from equation (14) that

$$F\left(\frac{\lambda}{\mu_h + \alpha_h}\right) = -\frac{\beta_h \beta_m N_m \lambda \alpha_h \left[\frac{\gamma\lambda + (1-\varphi)\lambda(\alpha_h+\mu_h)}{\nu_h(\alpha_h+\mu_h)} + \frac{\gamma\lambda}{\mu_h(\alpha_h+\mu_h)}\right]}{\beta_m\left(\lambda - \mu_h \frac{\lambda}{(\alpha_h+\mu_h)}\right) + \alpha_h(\alpha_m + \mu_m)} - \left[\frac{(\mu_h + \nu_h)(\gamma + \alpha_h + \mu_h)\left(\lambda - \frac{\mu_h \lambda}{\mu_h+\alpha_h}\right)}{\alpha_h \nu_h} - \frac{\lambda(\alpha\nu_h + \varphi)}{\nu_h}\right] < 0 \tag{15}$$

if

$$\frac{(\mu_h + \nu_h)(\gamma + \alpha_h + \mu_h)}{(\mu_h + \alpha_h)(\alpha\nu_h + \varphi)} > 0.$$

and $F\left(\frac{\lambda}{\mu_h}\right) = \alpha\lambda + \frac{\varphi\lambda}{\nu_h} > 0$

It would be sufficient if we show that $F(N_h) = 0$ has one and only one root.

From equation (14), it can be seen that $F\left(\frac{\lambda}{\mu_h+\alpha_h}\right) < 0$ and $F\left(\frac{\lambda}{\mu_h}\right) > 0$.

Hence there exists a root $\hat{N}_h$ of $F(N_h) = 0$ in $\frac{\lambda}{\mu_h+\alpha_h} < N_h < \frac{\lambda}{\mu_h}$.

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Also, $F^{j}(N_h) > 0$, provided

$$[N_h - \frac{(\gamma+\alpha_h+\mu_h)(\lambda-\mu_h N_h)-\alpha_h\varphi\lambda}{\alpha_h \nu_h} \alpha\ \nu_h\ _h - (\frac{\lambda-\mu_h N_h}{\alpha_h}) - g_h(N_h)] > 0$$

in $\frac{\lambda}{\mu_h+\alpha_h} < N_h < \frac{\lambda}{\mu_h}$.

Thus there exists a unique root of $F(N_h) = 0$ ($say \hat{N}_h$) in $\frac{\lambda}{\mu_h+\alpha_h} < N_h < \frac{\lambda}{\mu_h}$.

So the equilibrium point E_3 exists provided $\frac{(\mu_h+\nu_h)(\gamma+\alpha_h+\mu_h)}{(\mu_h+\alpha_h)(\alpha\nu_h+\varphi)} > 0$

4 Stability Analysis

In the following theorem, we discuss about the local stability of the three equilibrium points E_1, E_2 and E_3 calculated in the previous section.

Theorem 3. *The equilibrium points E_1 and E_2 are stable and the equilibrium E_3 is stable if $q_1q_2 - q_3 > 0$ and $q_1q_2q_3 - q_3^2 - q_4 > 0$.*

Proof. Let us consider the first equilibrium point $E_1(0, 0, 0, N_h^*, 0, 0)$ and the variational matrix M_1 corresponding to this point is given by

$$M_1 = \begin{pmatrix} -(\mu_h+\nu_h) & 0 & 0 & 0 & \beta_h N_h^* & 0 \\ \nu_h & -(\gamma+\alpha_h+\mu_h) & 0 & 0 & 0 & 0 \\ 0 & \gamma & -\mu_h & 0 & 0 & 0 \\ 0 & -\alpha_h & 0 & -\mu_h & 0 & 0 \\ 0 & 0 & 0 & 0 & -(\mu_m+\alpha_m) & 0 \\ 0 & 0 & 0 & 0 & 0 & -(\mu_m+\alpha_m) \end{pmatrix}$$

For the above matrix the characteristic polynomial is given

by $(\mu_h+\nu_h+\lambda)(\mu_h+\lambda)^2(\gamma+\alpha_h+\mu_h+\lambda)^2(\mu_m+\alpha_m+\lambda)^2 = 0$

It can clearly be seen that all the roots of λ are negative.

Hence equilibrium point E_1 is stable.

We consider the equilibrium point $E_2(0, 0, 0, N_h^{**}, 0, N_m^{**})$ and the variational matrix M_2 corresponding to this point is given by

$$M_2 = \begin{pmatrix} -(\mu_h+\nu_h) & 0 & 0 & 0 & \beta_h N_h^{**} & 0 \\ \nu_h & -(\gamma+\alpha_h+\mu_h) & 0 & 0 & 0 & 0 \\ 0 & \gamma & -\mu_h & 0 & 0 & 0 \\ 0 & -\alpha_h & 0 & -\mu_h & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -(\mu_m+\alpha_m) \\ 0 & \beta_m N_m^{**} & 0 & 0 & -(\mu_m+\alpha_m) & 0 \\ 0 & 0 & 0 & 0 & 0 & -(\mu_m+\alpha_m) \\ 0 & 0 & 0 & 0 & 0 & -(\mu_m+\alpha_m) \end{pmatrix}$$

The characteristic polynomial corresponding to the above matrix is

$$(\mu_m + \alpha_m + \lambda)(\mu_h + \lambda)^2(\lambda^3 + p_1\lambda^2 + p_2\lambda + p_3) = 0 \tag{16}$$

where

$p_1 = \gamma + \alpha_h + \nu_h + 2\mu_h + \mu_m + \alpha_m$

$p_2 = (\gamma + \alpha_h + \mu_h)(\mu_h + \nu_h + \mu_m + \alpha_m) + (\mu_m + \alpha_m)(\mu_h + \nu_h)$ p_3

$= (\mu_h + \nu_h)(\mu_m + \alpha_m)(\gamma + \alpha_h + \mu_h) - \beta_h\beta_m N_h^{**} N_m^{**} \nu_h$

The eigenvalues of (16) are $-\mu_h$, $-(\mu_m + \alpha_m)$ and the roots of the polynomial $\lambda^3 + p_1\lambda^2 + p_2\lambda + p_3$.

According to Routh-Hurwitz criterion, the cubic polynomial $\lambda^3 + p_1\lambda^2 + p_2\lambda + p_3$ will have negative real roots if $p_1p_2 - p_3 > 0$.

Here

$p_1p_2 - p_3 = (\mu_h + \nu_h)(\gamma + \alpha_h + \mu_h + \mu_m + \alpha_m)(\gamma + \alpha_h + 2\mu_h + \nu_h + \mu_m + \alpha_m) + (\mu_m + \alpha_m)(\gamma + \alpha_h + \mu_h)(\gamma + \alpha_h + \mu_h + \mu_m + \alpha_m) + \beta_h\beta_m N_h^{**} N_m^{**} \nu_h$

which is positive. Hence equilibrium point E_2 is stable.

Lastly, we consider the interior equilibrium point $E_3(\hat{E}_h, \hat{I}_h, \hat{R}_h, \hat{N}_h, \hat{I}_m, \hat{N}_m)$ and the variational matrix M_3 corresponding to it is,

$$M_3 = \begin{bmatrix} -(\beta_h\hat{I}_m + \mu_h + \nu_h) & -\beta_h\hat{I}_m & -\beta_h\hat{I}_m & \beta_h\hat{I}_m & \beta_h(\hat{N}_h - \hat{E}_h - \hat{I}_h - \hat{R}_h) & 0 \\ \nu_h & -(\gamma + \alpha_h + \mu_h) & 0 & 0 & 0 & 0 \\ 0 & \gamma & -\mu_h & 0 & 0 & 0 \\ 0 & -\alpha_h & 0 & -\mu_h & 0 & 0 \\ 0 & \beta_m(\hat{N}_m - \hat{I}_m) & 0 & 0 & -(\beta_m\hat{I}_h + \mu_m + \alpha_m) & \beta_m\hat{I}_h \\ 0 & 0 & 0 & 0 & 0 & -(\mu_m + \alpha_m) \end{bmatrix}$$

The characteristic polynomial corresponding to the above

matrix is, $(\mu_h + \lambda)(\mu_m + \alpha_m + \lambda)(\lambda^4 + q_1\lambda^3 + q_2\lambda^2 + q_3\lambda + q_4) = 0$

where

$q_1 = \beta_h\hat{I}_m + \beta_m\hat{I}_h + \gamma + \alpha_h + 3\mu_h + \nu_h + \mu_m + \alpha_m$

$q_2 = (\beta_m\hat{I}_h + \mu_m + \alpha_m)(\beta_h\hat{I}_m + \gamma + \alpha_h + 3\mu_h + \nu_h) + (\beta_h\hat{I}_m + \mu_h + \nu_h)\ (\beta_h\hat{I}_m\nu_h + \gamma + \alpha_h + \mu_h)$

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$q_3 = \mu_h(\beta_m\hat{I}_h + \mu_m + \alpha_m)(\beta_h\hat{I}_m + \gamma + \alpha_h + 2\mu_h + \nu_h) + (\beta_m\hat{I}_h + \mu_m + \alpha_m +$

$\mu_h)((\beta_h\hat{I}_m + \mu_h + \nu_h)(\gamma + \alpha_h + \mu_h) + \beta_h\hat{I}_m\nu_h) + \beta_h\hat{I}_m(\gamma + \alpha_h)\nu_h$

$q_4 = (\beta_m\hat{I}_h + \mu_m + \alpha_m)(\gamma + \alpha_h + \mu_h)[\mu_h(\beta_h\hat{I}_m + \mu_h + \nu_h) + \beta_h\hat{I}_m\nu_h)]$

$+\nu_h\beta_m(\hat{N}_m - \hat{I}_m)(\hat{N}_h - \hat{E}_h - \hat{I}_h - \hat{R}_h)$

The eigenvalues of above characteristic equation are $-(\mu_m + \alpha_m)$, $-\mu_h$ and the roots of the polynomial $\lambda^4 + q_1\lambda^3 + q_2\lambda^2 + q_3\lambda + q_4$.

By the Routh-Hurwitz criterion, the polynomial $\lambda^4 + q_1\lambda^3 + q_2\lambda^2 + q_3\lambda + q_4$ has roots with negative real part if $q_1q_2 - q_3 > 0$ and $q_1q_2q_3 - q_3^2 - q_4 > 0$

Hence, the equilibrium point E_3 is locally asymptotically stable if

$q_1q_2 - q_3 > 0$ and $q_1q_2q_3 - q_3^2 - q_4 > 0$. Hence

the theorem is proved.

5 Discussion

We formulated and analysed a system of non-linear ordinary differential equa- tions by considering the four compartments (susceptible, exposed, infected and recovered) for the human population and two compartments (susceptible and in- fected) for mosquitoes. The existence of the region in which the model is both epi- demiologically and mathematically well-posed has been found.The steady states and their stability was calculated for the defined model. It can be observed that due to the presence of infective immigrants, there is no disease-free equilibrium point for this model.

From the above calculations it can be seen that the fraction of the exposed or infected population do not play much role in the calculation of the non-trivial equilibrium points and analysing the stability. However, numerical simulation of the above model may throw some insight on the effect of infective immigrants on the model.

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