Estimation of Stability derivative of an Oscillating cone in Hypersonic Flow

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Abstract: Formulae for the Stiffness and Damping derivatives are obtained in a closed form in the present context with the assumptions that the gas is non-viscous and perfect, the motion is quasi-steady and quasi-axisymmetric, and the nose semi angle of the cone is such that the Mach number M_2 behind the shock $M_2 \geq 2.5$. Results are presented for cone for $\gamma = 1.4$,at different Mach numbers and semi angles of the cone. It is observed that the neutral point shifts away from the apex of the cone as semi angle increases. So is the case with the minima of the curves for damping derivative. Also it is seen that an increase in Mach number after 10 marginally contributes to any variations in the values of stiffness and Damping derivative which is in accordance with the Mach number independence Principle. These results are likely to find wide applications in high speed flow problems.

Keywords: **High Speed Flow, Hypersonic Flow, Oscillating cone, Stiffness derivative**

I. INTRODUCTION

The present work evaluates stability derivatives in pitch for non-slender axi-symmetric Ogives oscillating in hypersonic flow. At hypersonic speeds the "nose cones" are often non-slender and blunt. The reason for such a configuration, is the problem of aerodynamic heating and hence the heat transfer at such speeds. Although the present work is not for blunt bodies with detached shocks, once a theory is developed for the ogives with sharp nose, it can then be extended to more practical shapes to incorporate the bluntness. It is really interesting to note that the study of hypersonic flows, which was restricted to slender bodies and low angles of attack, should attain a stage of non-slender shapes and at large angle of attack flows; promising an emphasized development of efficient future hypersonic systems.

Ghosh([1]) developed a new hypersonic similitude with the assumptions of attached bow-shock and Mach number behind the shock being greater than 2.5. This similitude was valid for the windward surface of an aero-foil with large flow deflection. His work was extended to oscillating non-planar wedges by Crasta and Khan to calculate the aerodynamic pitching moment derivatives in both Supersonic ([2]) and Hypersonic flows ([3],[4]and[5]).

The large deflection similitude of Ghosh ([1]) has been extended by Ghosh, K.,([2]) to axisymmetric bodies with attached shock. Equivalence with a new piston-motion with axial symmetry has been established. The cone (or quasi-cone) results from the revolution of a wedge (or quasi-wedge) round the streamwise axis and a similar revolution of the independent fluid slab ([1]) produces an axially symmetric conico-annular space. It is shown by Ghosh, K.,([2]) that the flow past a cone or quasi-cone is equivalent to a piston-motion in this conico-annular space, which is called the similitudinal slab. Although Ghosh, K., ([2]) gives similitude for cones and quasi-cones, he gives a solution based on the similitude for a cone only. This solution gives a constant density shock layer. Hence the constant density form of the unsteady Bernoulli's equation is used to find pressure on the cone surface in terms of the piston Mach number.

Results are obtained for hypersonic flow for a perfect gas over oscillating cones and ogives of different Mach numbers and semi- angles.

II. ANALYSIS

The expression for the pressure ratio of a steady cone at zero incidence (Ghosh, K., 1984), provided the bow shock is attached, is

$$\frac{P_{bo}}{P_\infty} = 1 + \gamma M^2{}_{po}\left(1 + \frac{1}{4}\varepsilon\right) \quad (1)$$

Where the density ratio is

$$\varepsilon = \frac{2 + (\gamma - 1)M^2{}_{po}}{2 + (\gamma + 1)M^2{}_{po}} \quad (2)$$

And M_{po} is the piston Mach number of the equivalent piston, operating in a conico-annular space; P_{bo} is the pressure on the body (cone) surface at zero incidences.

$$M_{po} = M_\infty \sin\theta_c$$

Where θ_c is the cone semi angle.

Now

$$\frac{dP_{bo}}{dM_{po}} = 2\gamma P_\infty M_{po}\left[1 + \frac{1}{4}\left(1 + \frac{1}{2}M_{po}.\frac{d\varepsilon}{dM_{po}}\right)\right] \quad (3)$$

Where

$$\frac{d\varepsilon}{dM_{po}} = \frac{-8M_{po}}{\left[2 + (\gamma + 1)M^2{}_{po}\right]^2} \quad (4)$$

Thus using the definition of Stiffness and damping derivative and using the above, the expression for Stiffness derivative is obtained as

$$C_{m_\theta} = D\left[h^3(1 - 2n^2) - (1 - h)\left\{H(2 + h) + n^2 h(1 + 2h)\right\}\right] \quad (5)$$

And the expression for damping derivative is obtained as

$$C_{m_q} = \frac{D}{2}\left[h^4(2n^2 - 3n^4 - 1) - (1 - h)\left\{H(3H + h(H + 2n^2) + 2h^2 n^2) + n^4 h^2(1 + 3h)\right\}\right] \quad (6)$$

Where

$$D = \frac{2}{3(1 + n^2)}\left[1 + \frac{1}{4}\left(\varepsilon + \frac{1}{2}K\frac{d\varepsilon}{dM_{po}}\right)\right] \quad (7)$$

And

$$H = \left(1 - h + n^2\right)$$

Various results have been plotted and discussed.

III. RESULTS AND DISCUSSION

The deviance of Stiffness derivative with pivot position for Mach number M=5 is depicted in Figure1. It is noted that Stiffness derivative decreases with pivot position and also with semi vertex angle. There is a 32%,19%,15% decrease in the magnitude of Stiffness derivative whenever semi vertex angle shows an increment from 5 to 25 degrees.

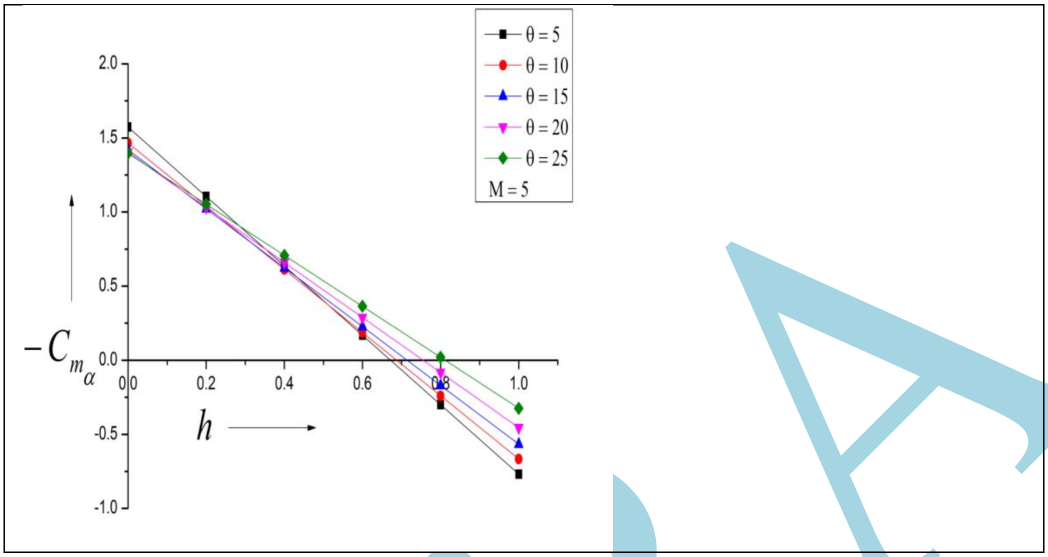

Figure 1: Variation of Stiffness derivative with h for M=5.

The deviance of Stiffness derivative with pivot position for Mach number M=7 is depicted in Figure 2.The same trend as above is seen. Further there is a decrement in the magnitude of Stiffness derivative when there is an increment in Mach number.

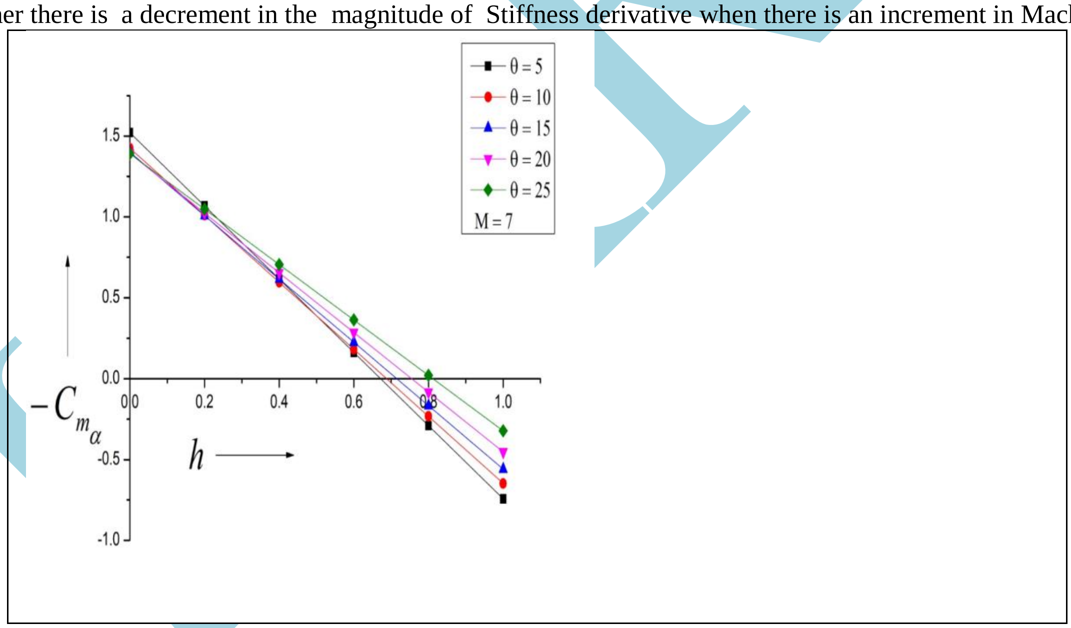

Figure 2: Variation of Stiffness derivative with h for M=7

The deviance of Stiffness derivative with pivot position for Mach number M=9 is depicted in Figure 3. The trend is on the similar lines as discussed above.

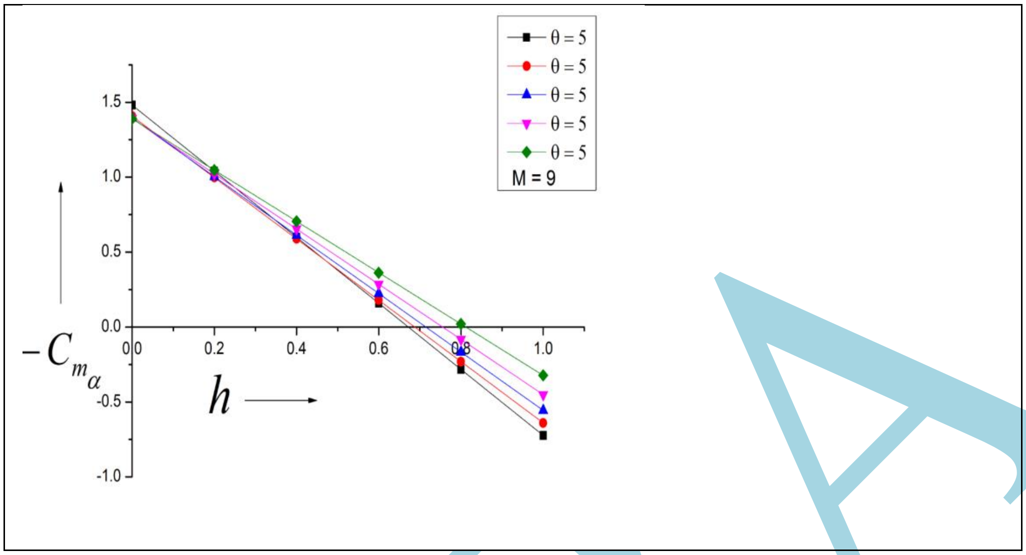

Figure 3: Variation of Stiffness derivative with the h for M=9.

The deviance of Stiffness derivative with pivot position for Mach number M=5 is depicted in Figure 4. The magnitude of stiffness derivative is decreased considerably as compared to that for Mach number 9. The reason for this trend may be due to the shock strength when Mach number is increased and also, due to this change in Mach number the pressure on the surface will get modified considerably. The Stiffness derivative shows linear decrement with pivot position as well as with semi vertex angle as it is evident from the expression.

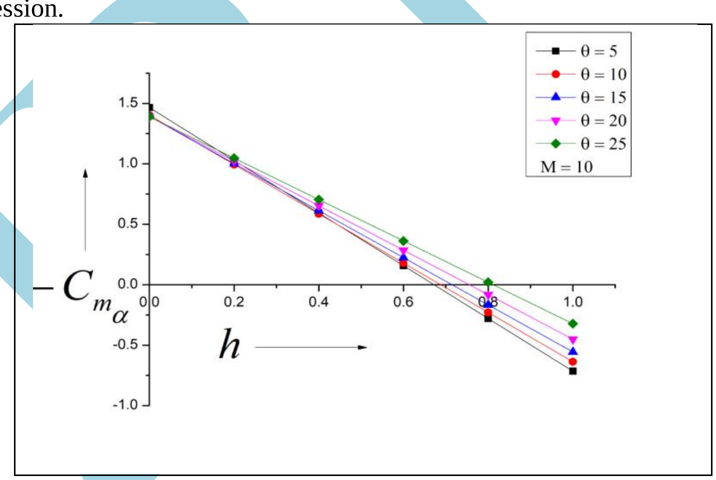

Figure 4: Variation of Stiffness derivative with the h for M=10

The deviance of Stiffness derivative with pivot position for Mach number M=5 is depicted in Figure .It is seen that an increase in Mach number from 10 to 15 rarely contributes to any change in the values of Stiffness derivative which is in accordance with the Mach number independence principle.

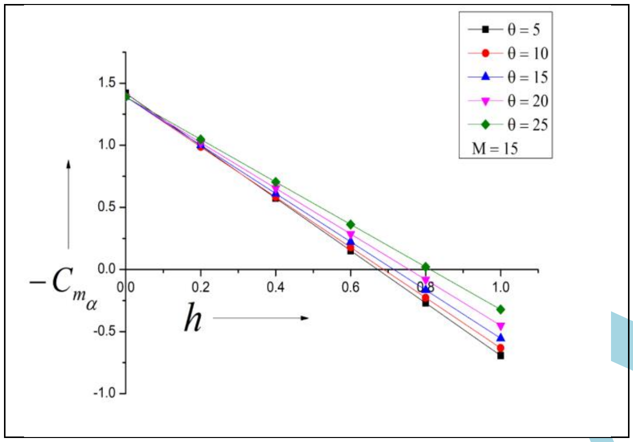

Figure 5: variation of Stiffness derivative with h for M=15

Figure 6 depicts the initial reduction in the value of damping derivative with pivot position and the attaining a minima and further with the rise in the value of Damping derivative with pivot position for semi angle 5 degrees. A minima is been attained in the range from 60 % to 70% from the nose of the cone.

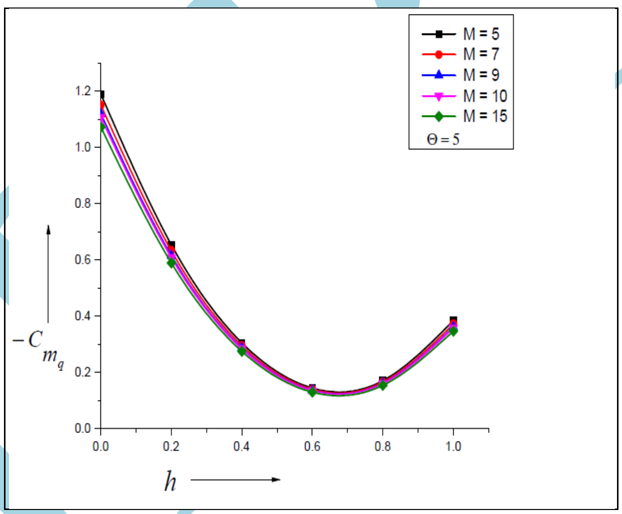

Figure 6: Damping derivative variation with pivot position for semi angle 5 degrees.

Figure 7 depicts the variation of damping derivative with pivot position for semi angle 10 degrees. It can be evidently noticed that as semi angle increases from 5 to 10 degrees the value of damping derivative is reduced from 1.2 to 1.1 at the nose of the cone and further the point of attainment of minima being shifted from 65 % to 70 % from the nose of the cone,for otherwise the trend being similar to the above figure.

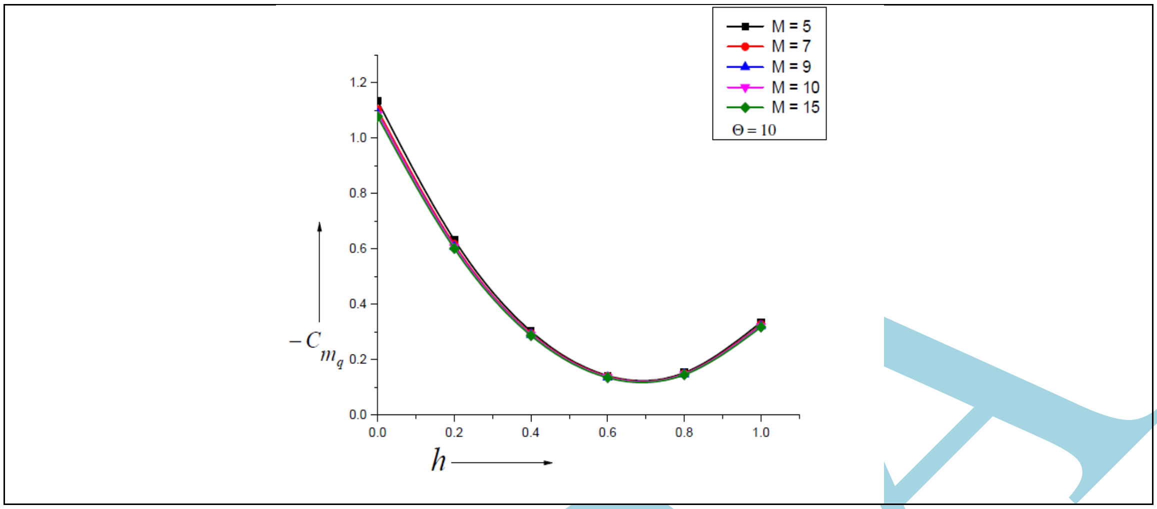

Figure 7: Damping derivative variation with pivot position for semi angle 10 degrees.

Figure8 depicts the variation of damping derivative with pivot position for semi angle 15 degrees. It is observed from the figure that for semi angle 15 degrees there is no dependency on Mach number for the damping derivative variation with pivot position, otherwise the trend being similar as discussed above.

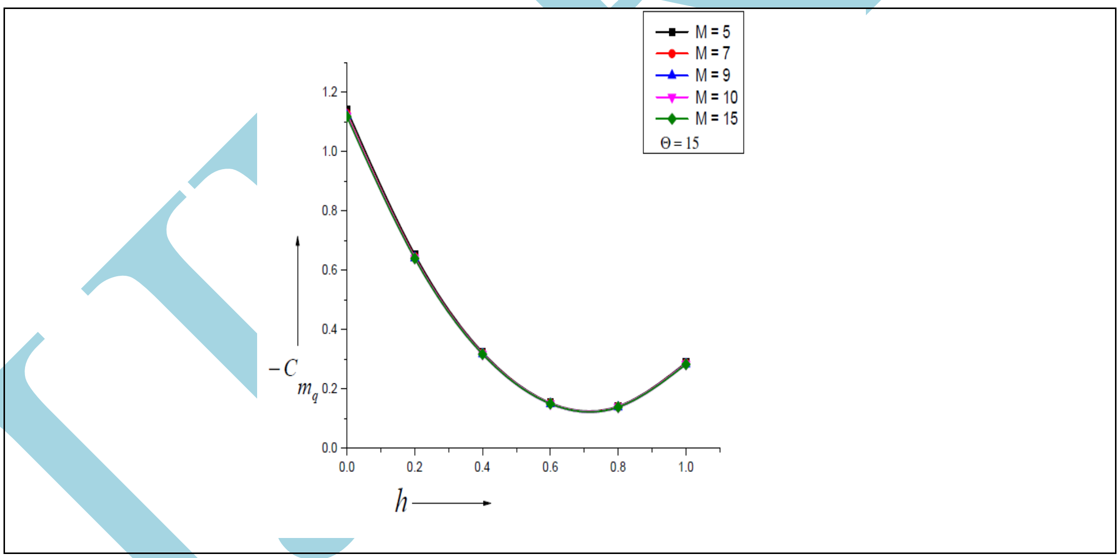

Figure 8: Damping derivative variation with pivot position for semi angle 15 degrees.

Figure 9 depicts the damping derivative variation with pivot position for semi angle 20 degrees. The trend is on the similar lines as discussed above.

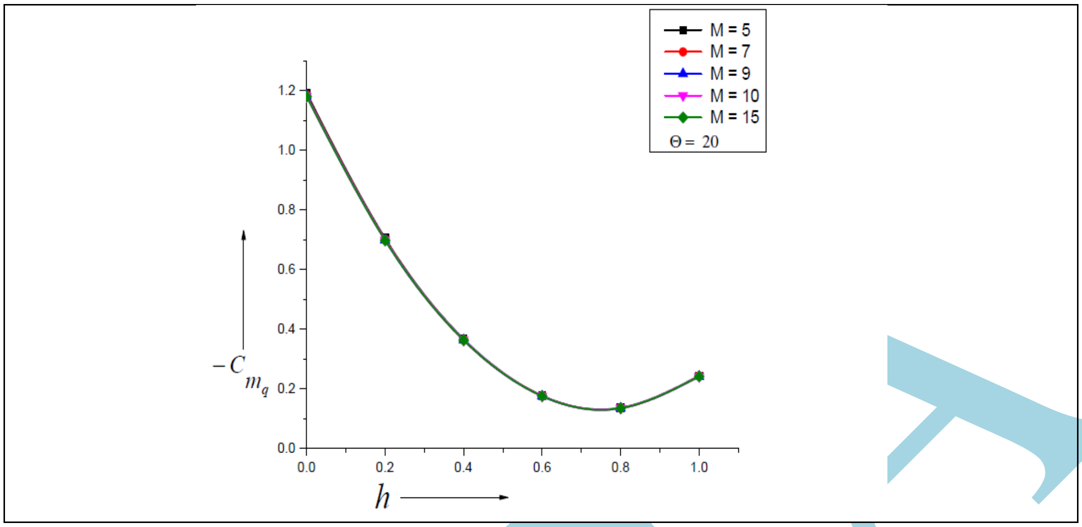

Figure 9: Damping derivative variation with pivot position for semi angle 20 degrees.

IV. Conclusion

The present theory for Oscillating cone is restricted to quasi-steady case. The main difficulty encountered in treating the unsteady case would be due to the flow being unsteady as well as non-uniform in the conical– annular space even for a steady piston. Viscous effects have been neglected. The expressions so obtained for stability derivative in pitch are valid for a slender ogive which often approximates to the whole fuselage of an aircraft. The Stiffness derivative shows linear decrement with pivot position as well as with semi vertex angle as it is evident from the expression. An increase in Mach number from 10 to 15 rarely contributes to any change in the values of Stiffness derivative which is in accordance with the Mach number independence principle. The value of Damping derivative initially reduces with pivot positions attains a minima and then increases.

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