Predictive modeling of suddenly expanded flow process in the Supersonic Mach number regime using response surface methodology

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Abstract: **The present work uses design of experiments (DOE) technique along with response surface methodology to develop linear models, to establish linear input-output relationships in a suddenly expanded flow process. Mach number (M), nozzle pressure ratio (NPR), area ratio (AR) and length to diameter (L/D) ratio have been considered as the input parameters, which controls the output (i.e. base pressure). Full factorial DOE has been implemented for developing the linear model. Experiments were conducted to measure base pressure by two means i.e. without control (WoC) and with the use of active control (WC). The adequacy of developed models was checked through statistical analysis. Fifteen random test cases were conducted in order to validate the models. It is observed that, both linear regression models for base pressure without and with control are statistically adequate and capable of making accurate predictions.**

***Keywords*: design of experiments, base pressure, Mach number, nozzle pressure ratio, area ratio and length to diameter ratio.**

I. INTRODUCTION

Flow separation in the field of base flow aerodynamics is said to be a matter of prime concern in aerodynamic vehicles such as missiles, rockets and projectiles. The main reason for this being formation of low pressure circulation region very near to its base. The pressure in this region is expected to be significantly lower when compared to the atmospheric pressure (Khan et al 2006). This difference in pressure leads to the formation of base drag which can be upto two-third of the total drag on a revolutionary body. Many active and passive controls have been used in the earlier studies for reducing base drag and favorable results have been achieved thereby. However, very few studies have been carried out by use of active control. Flow separation is a complex phenomenon which is generally characterized by flow field of abrupt axisymmetric expansion. This type of flow field is divided by a shear layer into two regions, one being the region of main flow and the other being the recirculation flow region. The point where the dividing streamlines strike the wall is called as the reattachment point. The main features of suddenly expanded flow field are shown in Fig. 1.

(Korst, 1956) studied the problem of base pressure for transonic and supersonic flow cases. A physical flow model was developed based on interactions between shear flow, adjacent free stream and conservation of mass in the wake. (Khan et al. 2002) carried out experiments to study the effect of micro jets in an axisymmetric duct. The study was conducted for Mach numbers of 2.0, 2.5 and 3.0 and for a level of overexpansion of (P_e/P_a=0.277). The outcome of the work identified a suitable value of L/D ratio for increase or decrease of base pressure for a given value of Mach number and nozzle pressure ratio. (Khan et al. 2003) studied variation of base pressure for Mach numbers 1.87, 2.2 and 2.58. The studies were carried out for nozzle pressure ratios of 3, 5, 7, 9 and 11. An increase of upto 95 percent in base pressure was observed for certain set of combination of parameters. (Khan et al. 2004) studied base pressure variation by use of micro jets for Mach numbers 1.25, 1.3, 1.48, 1.6, 1.8 and 2.0. The experiments were conducted for a level of under expansion of (P_e/P_a=1.5). The studies found micro jets to be effective whenever nozzles were underexpanded. (Khan et al. 2004) investigated the micro jets effect for suddenly expanded flows for nozzles that were subjected to correct expansion. It was found that the microjets were not effective for Mach numbers 1.25, 1.3, 1.48, 1.6, 1.8 and 2.0. The base pressure values experienced a marginal change. Control effectiveness for base pressure values in a suddenly expanded axisymmetric duct for Mach numbers 1.25, 1.3, 1.48, 1.6, 1.8, 2.0, 2.5 and 3.0 was studied by (Khan et al. 2006). The experiments were conducted for nozzle pressure ratios of 3, 5, 7, 9 and 11. It was concluded that the values of base pressure increased with increase in area ratio for a given Mach number, L/D ratio and nozzle pressure ratio.

(Baig et al. 2001) studied control of base flows with micro jets in a suddenly expanded flow process for Mach numbers of 1.87, 2.2 and 2.58. The NPRs were varied from 3 to 11 in steps of 2 and lengths to diameter ratios from10 to 1 were employed. A 65 percent increase in base pressure was achieved for certain combination of parameters of the study. (Jaimon et al. 2016) conducted experiments to study the variation of base pressure for Mach number 3.0, area ratio of 4.84 and NPRs of 3, 5, 7, 9 and 11.

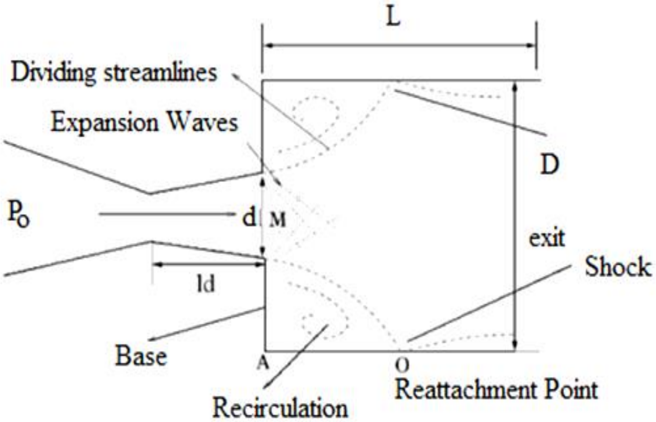

Figure 1: Suddenly expanded flow field.

Accordingly L/D ratio was considered from 10 to 1 where tests were conducted for L/D 10, 9, 8, 7, 6, 5, 4, 3, 2 and 1. The results showed that for increase in the NPR values once the flow is attached to the duct, the base pressure progressively decreased. Based on the literature cited above, even though there is considerable amount of literature available on the problem of sudden expansion, the studies provide a very generic and graphical approach towards variation of output base pressure with respect to the input parameters. Therefore, the present study aims to study variation of base pressure by use of design of experiments as per full factorial design to provide insights into the individual contribution of each input parameter affecting base pressure by development of linear statistical models.

II. METHODOLOGY

Selection of variables and their levels

The principle properties of base pressure are generally governed by Mach number and nozzle pressure ratio. The reason for this is due to the level of expansion which significantly influences the base pressure during the flow process. It is also to be noted that area ratio and L/D ratio play a significant role in affecting base pressure and has been mentioned in the previous studies. In this work, the following parameters are considered for experimentation:

i. Mach number
ii. Nozzle pressure ration (NPR)
iii. Area ratio (AR)
iv. Length to diameter ratio (L/D)

The levels of the variables were set by conducting a few trial experiments and a detailed literature survey on the factors affecting base pressure. The parameters and their levels are listed in Table 1.

Table 1- Factors and their respective levels

Process Parameters		Levels	
Description	Code	Low (-1)	High (1)
Mach number	A	2.0	3.0
NPR	B	5	9
Area Ratio (AR)	C	3.25	6.25
L/D Ratio	D	4	8

Conducting experiments

The following steps were involved in conducting the experiments:

i. Design the nozzle for Mach 2.0, 2.5 and 3.0 as per Genick et al. 2007.
ii. Measure the base pressure for the cases of without and with active control.
iii. Conducting experiments with different combinations of parameters as per the full factorial design shown in Table 2.
iv. Conducting experiments with randomly generated 15 combinations of the process parameters for testing the performances of the developed models.

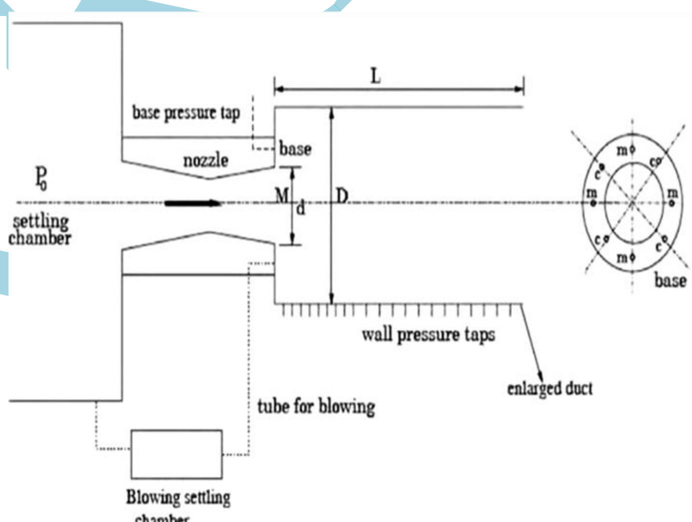

Fig. 2: Experimental setup

Developing linear models

In the present paper, full factorial design was used to develop linear models for the responses such as base pressure without and with the use of active control (micro jets).

Performing statistical analysis of data and comparison of models

Analysis of variance (ANOVA) was performed for each of the responses separately, to test the adequacy of the models. The models were compared response-wise for 15 test cases for validation.

III. EXPERIMENTATION

Fig. 2 shows the experimental setup used for the present study. The nozzles are machined in such a way that, there are eight holes stationed at the exit periphery of it. Out of these, four holes are marked (c) which have been used for blowing purpose and the remaining four marked (m) used for base pressure (P_b) measurement. Base pressure was controlled by blowing through the control holes (c), using pressure from a settling chamber by employing a tube connecting the settling chamber and the control holes.

Table 2- Design matrix for full factorial design

Sample no.	Input parameters				Responses	
	A	B	C	D	P_b/P_a (WoC)	P_b/P_a (WC)
1	-1	-1	-1	-1		
2	+1	-1	-1	-1		
3	-1	+1	-1	-1		
4	+1	+1	-1	-1		
5	-1	-1	+1	-1		
6	+1	-1	+1	-1		
7	-1	+1	+1	-1		
8	+1	+1	+1	-1		
9	-1	-1	-1	+1		
10	+1	-1	-1	+1		
11	-1	+1	-1	+1		
12	+1	+1	-1	+1		
13	-1	-1	+1	+1		
14	+1	-1	+1	+1		
15	-1	+1	+1	+1		
16	+1	+1	+1	+1		

A PSI system 2000 make pressure transducer has been used for the measurement of base pressure values. It had 16 channels with digital display pressure readings ranging from about 0-300psi. The readings were displayed at an average of 250 samples per second. The base pressure measured (P_b) along the duct was non-dimensionalized by dividing the value by atmospheric pressure (P_a).

IV. RESULTS AND DISCUSSIONS

Experimental data were collected as per full factorial design (2-level). The data that was collected was analyzed separately using regression analysis and response surface methodology. The data was then subjected to significance tests, model validation by conducting a few random experiments within the ranges of respective parameters.

Model development and statistical analysis

The experimental data collected was used to develop linear models for the different responses. Response-wise models and their statistical analysis are given below.

Response- Base pressure (WoC)

The following linear models have been developed based full factorial design for the response -base pressure without the use of active control (WoC). The input output relationships have been derived using the collected experimental data which was later implemented into the commercially licensed MINITAB software. They are expressed in coded from in Eq. (1).

$$(P_b/P_a)_{FFD} = -6.8888 + 2.66991\,A + 0.64271\,B + 1.51400\,C + 0.33907\,D - 0.22949\,AB - 0.51763\,AC - 0.11265\,AD - 0.167833\,BC - 0.013216\,BD - 0.094073\,CD + 0.057958\,ABC + 0.000904\,ABD + 0.032406\,ACD + 0.007427\,BCD - 0.002177\,ABCD \quad 1)$$

Significant tests were carried out for the different models separately and the significant terms were identified. Moreover, the coefficients of multiple correlations were determined to test the accuracy of the model. Table 3 shows the coefficient of multiple correlation values and the insignificant terms of different models for the response – base pressure (WoC). ANOVA tests have been conducted to check the adequacy of the model and are shown below in Table 4. It can be clearly noticed that the P-values for main effects, 2-way interactions, 3-way interactions and 4-way interactions are found to be less than 0.05 (corresponding to the 95% confidence level). It indicates that the above mentioned terms of linear model were found to be significant.

Response- Base pressure (WC)

The response equation developed for base pressure (WC) based on full factorial design is expressed in Eq. (2) as

$$(P_b/P_a)_{FFD} = -5.9872 + 2.25715\,A + 0.53340\,B + 1.35042\,C + 0.19030\,D - 0.17694\,AB - 0.44058\,AC - 0.04941\,AD - 0.147083\,BC + 0.007633\,BD - 0.067969\,CD + 0.047750\,ABC - 0.007979\,ABD + 0.021125\,ACD + 0.003719\,BCD - 0.000583\,ABCD \quad (2)$$

The ANOVA test was performed and the coefficients of correlations were determined to test statistical adequacy of the models. Table 3 shows the coefficients of correlation for the different models and their insignificant terms. The coefficients ofmultiple correlations were found to be close to 1.0, which indicate that all models fit well to the assumed response equations. Table 5 shows the results of ANOVA test conducted on the linear model, for the response- base pressure (WC). The P values shown in this table indicate that all the terms are significant.

Model testing

In the present section, the performances of the models developed have been validated response wise by conducting fifteen random experimental runs as shown in Table 5. The predicted values of the responses obtained through full factorial design were compared with their target (experimental) values. The line of best fit is used to make comparison. Here the measured values are compared with the model predicted values. It is observed the model has performed slightly better for base pressure (WC) when compared to base pressure (WoC) as all the data points are closely associated to the ideal line i.e. y=x line thereby indicating better prediction (Fig. 3). The values of percentage deviation are found to lie in the range of -17.25% to +12.34% for base pressure (WoC) and -12.63% to +12.74% for base pressure (WC) and is shown in Fig. 4. For base pressure (WoC) most of the data points lie away from the reference line whereas for base pressure (WC) the data points lie very close to the reference line. It is also important to note that FFD model has shown better prediction, in terms of average absolute percent deviation, for base pressure (WC) (see Fig. 5).

Table 3- Coefficient of multiple correlations and insignificant terms of different models for the response –base pressure (WoC)

Base Pressure	**Coefficient of correlation with all terms R**	**Coefficient of correlation without insignificant terms**	**Insignificant terms**
WoC	0.9995	0.9997	Nil
WC	0.9992	0.9996	Nil

Table 4- Results of ANOVA- Base pressure (WoC).

Source	**DF**	**Adj SS**	**Adj MS**	**F-Value**	**P-Value**
Main effects	4	0.39462	0.098656	116925.31	0.000
2-way interactions	6	0.33506	0.055843	66184.40	0.000
3-way interactions	4	0.27629	0.069072	81863.06	0.000
4-way interactions	1	0.00273	0.002730	3235.63	0.000
Residual Error	48	0.00004	0.000001		
Pure Error	48	0.00004	0.000001		
Total	63	4.11444			

Table 4- Results of ANOVA- Base pressure (WoC).

Source	**DF**	**Adj SS**	**Adj MS**	**F-Value**	**P-Value**
Main effects	4	0.37343	0.093358	149372.03	0.000
2-way interactions	6	0.34689	0.057814	92503.00	0.000
3-way interactions	4	0.27357	0.068393	109428.50	0.000
4-way interactions	1	0.00020	0.000196	313.60	0.000
Residual Error	48	0.00003	0.000001		
Pure Error	48	0.00003	0.000001		
Total	63	3.90758			

Model testing

In the present section, models of base pressure responses have been validated by conducting fifteen random experimental runs as shown in Table 5. The predicted values of the responses obtained through full factorial design were compared with the corresponding experimental values. The line of best fit is used to make comparison. Here the measured values are compared with the model predicted values. It is observed the model has performed slightly better for base pressure (WC) when compared to base pressure (WoC) as all the data points are closely associated to the ideal line i.e. y=x line thereby indicating better prediction (Figure 3). The prediction values for percent deviation were found to lie between -17.25% to +12.34% for base pressure (WoC) and -12.63% to +12.74% for base pressure (WC) and are shown in Figure 4. For base pressure (WoC) most of the data points lie away from the reference line whereas for base pressure (WC) the data points lie very close to the reference line. It is also noticed that FFD model showed improved prediction, for average absolute percent deviation, for base pressure (WC) (see Figure 5).

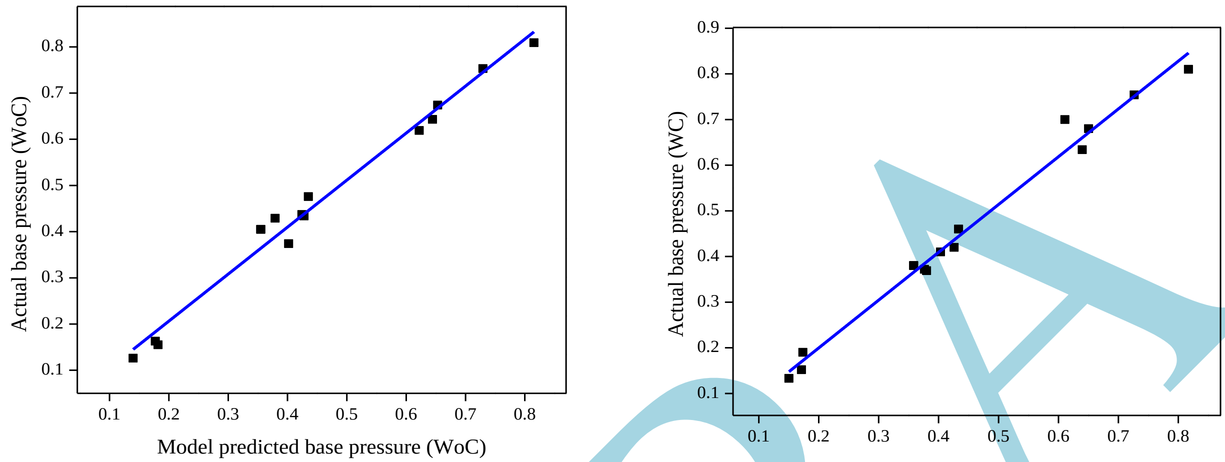

Figure 4. Standard deviation in prediction for FFD models developed for responses base pressure (WoC) and (WC)

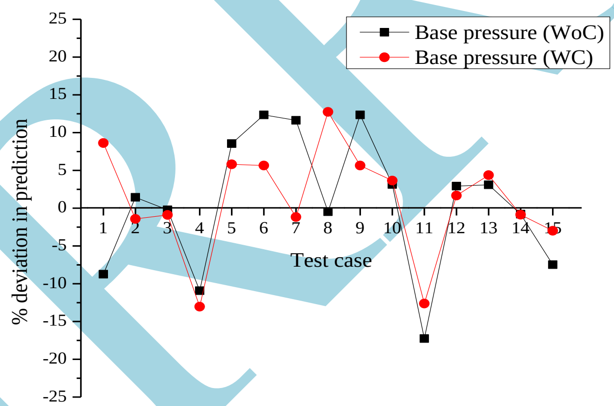

Figure 3. Comparison of model predicted base pressure with actual base pressure a) base pressure (WoC), b) base pressure (WC).

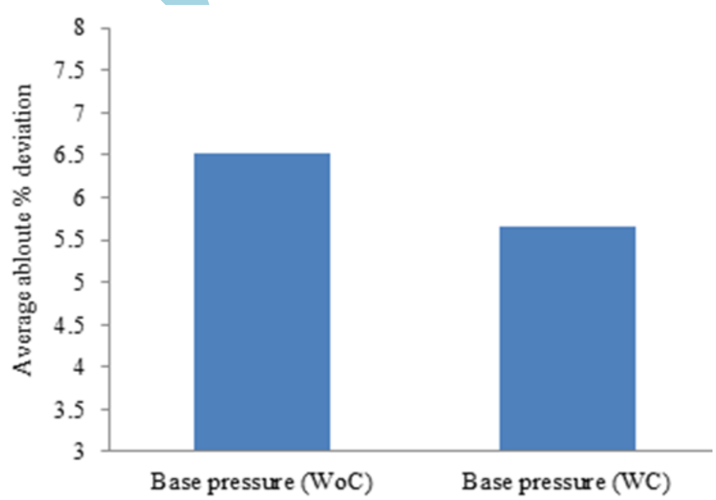

Figure 5. Comparison of average absolute percent deviation for responses of base pressure (WoC) and base pressure (WC).

Table 5-Random test cases and response values for model validation

S. I. No.	Mach number	NPR	Area ratio	L/D ratio	P_b/P_a (WoC)	P_b/P_a (WC)
1	2.0	9	3.25	6	0.163	0.19
2	2.5	5	3.25	6	0.434	0.42
3	3.0	7	3.25	4	0.643	0.634
4	2.0	5	3.25	5	0.126	0.133
5	2.5	5	3.25	5	0.476	0.46
6	2.0	5	4.75	6	0.405	0.38
7	2.5	9	4.75	5	0.429	0.372
8	3.0	9	4.75	4	0.619	0.7
9	2.0	5	4.75	8	0.369	0.37
10	3.0	5	4.75	8	0.753	0.754
11	2.0	9	6.25	5	0.155	0.152
12	2.5	9	6.25	6	0.437	0.41
13	2.5	5	6.25	8	0.674	0.68
14	3.0	5	6.25	6	0.809	0.81
15	2.5	9	6.25	8	0.374	0.369

V. CONCLUSIONS

Modelling has been carried out in order to develop linear models for a suddenly expanded flow process by full factorial design of experiments. The models have been developed for base pressure without active control and base pressure with active control respectively. ANOVA tests were carried out to inspect the statistical adequacy of the developed models for both the responses. It is observed that the models for both the base pressure responses are found to be statistically adequate. It is also important to note that the FFD model performed better for the case of base pressure with control when compared to without control in terms of absolute average percentage deviation. The regression models can be used to predict the response values without conducting experiments for the set of process parameters.

REFERENCES

[1]. Wick, R. S. (1953). The effect of boundary layer on sonic flow through an abrupt cross-sectional area change. Journal of the Aeronautical Science, 20, 675-682.

[2]. Quadros J. D., Khan S. A., Antony A. J., & Vas J.S. (2016). Experimental and numerical studies on flow from axisymmetric nozzle flow with sudden expansion for Mach 3.0 using CFD, *International Journal of Energy Environment and Economics*, 24(1), 87-99.

[3]. Rathakrishnan E., (2001). Effect of ribs on suddenly expanded flows. AIAA Journal, 39(7),1402- 1404.

[4]. Parappagoudar, M. B., Pratihar, D. K., & Datta, G. L. (2007) Linear and non-linear statistical modelling of green sand mould system International Journal of Cast Metals Research, 20(1), 1-13.

[5]. Korst, H. (1956). A theory of base pressure in transonic and supersonic flow, *Journal of Applied Mechanics* 23, 593-600.

[6]. Khan, S. A., E. Rathakrishnan (2002). Active control of suddenly expanded flows from over expanded Nozzles. *International Journal of Turbo and Jet Engine*, 19(1-2), 119-126.

[7]. Khan, S. A., E. Rathakrishnan (2003). Control of suddenly expanded flows with micro jets. *International Journal of Turbo and Jet Engines*, 20(2), 63-81

[8]. Khan, S. A., E. Rathakrishnan (2004). Active control of suddenly expanded flow from under expanded nozzles. *International Journal of Turbo and Jet Engines*, 21(4), 233-253.

[9]. Khan, S. A., E. Rathakrishnan (2004). Control of suddenly expanded flow from correctly expanded nozzles. *International Journal of Turbo and Jet Engines*, 21(4), 255-278.

[10]. Khan, S. A., E. Rathakrishnan (2006). Control of suddenly expanded flows, *Aircraft Engineering and Aerospace Technology: An International Journal*, 78(4), 293-309.

[11]. Genick, B. M. (2007). Gas Dynamics Tables, Version 1.3.

[12]. Baig, M. A. A., Al-Mufadi, F., Khan, S. A., & E. Rathakrishnan (2011). Control of base flows with micro jets. *International Journal of Turbo and Jet Engines*, 28 (4), 259-269.