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# Estimation of Stability Derivatives of a wedges at Supersonic Mach Numbers

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***Abstract:*** **The present study aims at to determine the stability derivatives of a wedge in pitch with attached shock at different angle of attack in supersonic flow. The stability is obtained for a wedge by using the concept of a piston moving in a cylinder at any arbitrary speed, which is dependent on the Mach number and the wedge semi vertex angle. From the results it is seen that as the semi-vertex angle of the wedge increases, the Stiffness derivative increases linearly and damping derivative first decreases, attains a minima and then increases linearly which results due to the change in pressure field and varying position of center of pressure from leading edge of the wedge. Also as Mach number increases, both Stiffness derivatives and damping derivatives assumes lower values at nose of the wedge. Real gas effects, viscous effects, bluntness of leading edge, and secondary wave effects have not been considered in the present study.**

***Keywords:*** **Angle of attack, Mach number, Stability derivatives, supersonic flow**

## I. INTRODUCTION:

Exact solution for 2-D flow of an oscillating wedge is given by Carrier [2] and Hui [8] obtained solution for an oscillating flat plate. This is applicable for supersonic flow with different angle of attack and wedge semi vertex angle when the leading edge of the obstacle is attached to the shock wave. Carrier [2] and Van Dyke [4] theory can be applied for both thick and slender oscillating wedges in which the disturbance in flow field is divided in to conservative and rotational field. Carrier [1, 2] found the Pressure disturbance around the wedge by considering rotational part of the flow partially. This concept is useful for supersonic flow, since the rotational part of the field is very small and it may differ for hypersonic flow.

For an oscillating wedge Light hill [3] "piston theory" and Miles [6] "shock expansion theory" has been considered, the effect of secondary wave reflected from bow shock has been neglected. For supersonic flow Pugh [7] obtained experimental results which can be used to compare theoretical results for hypersonic flow.

Strip theory used by Hui [8] to find the stability of an oscillating flat wing in Supersonic/hypersonic flow. The stability derivatives of the oscillating wing depends upon angle of attack, sweep angle, Mach number, specific heat ratio, but in his case the shock wave may be attached or detach from leading edge .

Crasta A and Khan [9, 10, 11] obtained supersonic flow past planar wedge, non-planar wedge and delta wing with curved leading edges. The stability derivatives of Roll and pitch of a delta wing has been acquired by Crasta A and Khan [12], whose leading edge is straight and curved.

## II. ANALYSIS

The Piston Mach number is given by

$$M_P = M_\infty \sin\alpha + \frac{q(x - x_0)}{a_\infty} \qquad (1)$$

Pressure ratio for Supersonic flow is given by

$$\frac{P}{P_\infty} = 1 + A\left(\frac{M_P}{\cos\phi}\right)^2 + A\left(\frac{M_P}{\cos\phi}\right)\sqrt{B + \left(\frac{M_P}{\cos\phi}\right)^2} \qquad (2)$$

Where, $A = \frac{\gamma(\gamma+1)}{4}$ $\quad B = \left(\frac{4}{\gamma+1}\right)^2$

Where $P_\infty$ stands for pressure on Leeward surface and P stands for pressure on windward surface. The co-efficient of 'α' in nose down restoring moment

$$-m\int_0^L (x - x_0)P\,dx \qquad (3)$$

gives the required Aerodynamic stiffness derivative

$$-c_{m_\alpha} = \frac{(\gamma+1)\tan\theta}{\cos^2\phi}\left[2 + \frac{\sqrt{M_\infty^2\sin^2\theta + \left(\frac{4}{\gamma+1}\right)^2\cos^2\phi}}{M_\infty\sin\theta} + \frac{M_\infty\sin\theta}{\sqrt{M_\infty^2\sin^2\theta + \left(\frac{4}{\gamma+1}\right)^2\cos^2\phi}}\right]\left[\frac{1}{2} - h_0\cos^2\theta\right] \qquad (4)$$

Aerodynamic damping derivative is given by,

$$-c_{m_q} = \frac{(\gamma+1)\tan\theta}{\cos^2\theta\cos^2\phi}\left[2 + \frac{\sqrt{M_\infty^2\sin^2\theta + \left(\frac{4}{\gamma+1}\right)^2\cos^2\phi}}{M_\infty\sin\theta} + \frac{M_\infty\sin\theta}{\sqrt{M_\infty^2\sin^2\theta + \left(\frac{4}{\gamma+1}\right)^2\cos^2\phi}}\right]*\left[\frac{1}{3} - h_0\cos^2\theta + h_0^2\cos^4\theta\right] \qquad (5)$$

To prove the expression for stiffness and damping derivative Leibnitz Rule for differentiation under integral sign concept has been used and some of the results are discussed in the section to follow

## III. RESULTS AND DISCUSSIONS:

Variation of stiffness derivatives with respect to pivot position by varying Mach number from M = 2 to M = 4.5 for various semi-vertex angles of the wedge are shown in Figs. 1 to 6. In figure 1 the results are shown up to semi-vertex angle 20 degrees. It is observed that with the progressive increase in the semi-vertex angle there increase in the stiffness derivatives as well due to the increase in the plan-form area of the wedge. For Mach 2.0 the maximum semi vertex angle taken is 20 degrees as to apply the theory the essential condition is that the shock must be attached with the nose of the wedge. Also, it is seen that the variations in the center of pressure is with in h = 0.5 to 0.56 and this shift in the center of pressure is because change in pressure distribution. Fig. 2 presents results for Mach 2.5, in this case the semi-vertex angle considered is 25 degrees and in view of this increased value of the semi-vertex angle, there is further shift of the center of pressure towards the trailing edge. However, the magnitudes are in the same range as that of in figure 1. Similarly figure 3 shows results for Mach 3 and at this Mach number the semi-vertex angle considered is 30 degrees. It is seen that due to further increase in the semi-vertex angle the center of pressure has further shifted towards the trailing edge with enhanced magnitude in the stiffness derivative. Similar results are seen in Figs. 4 to 6 at Mach 3.5, 4, and 4.5. At higher Mach number the magnitude of the stiffness derivative is decreasing linearly as Mach number increases. However, at higher Mach numbers there is no shift in the center of pressure, it remains constant for Mach

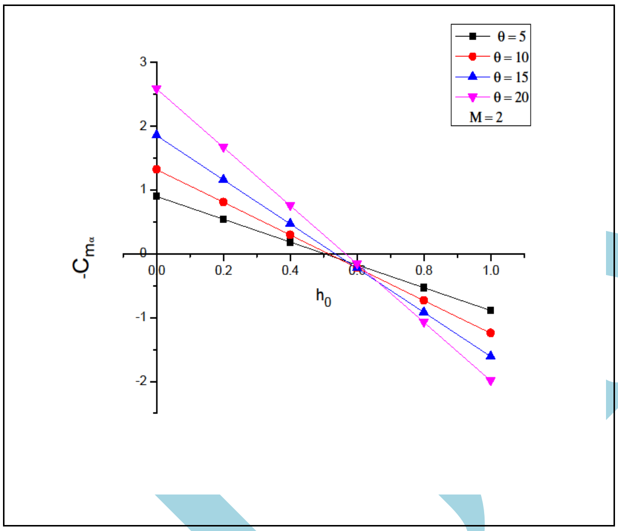


**Fig. 1 Stiffness Derivative Vs pivot at M = 2**

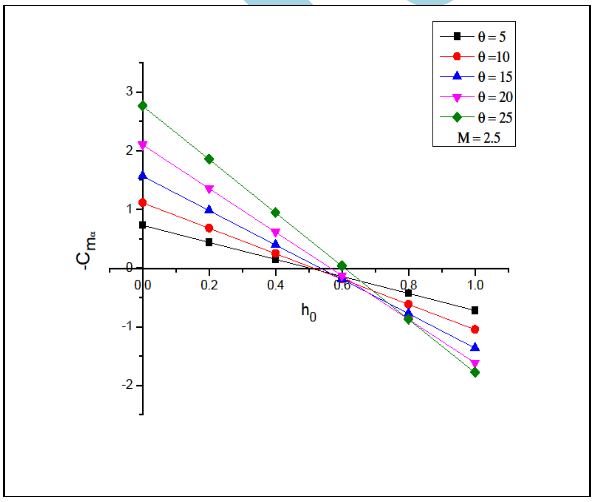


**Fig. 2 Stiffness Derivative Vs pivot position at M=2.5**

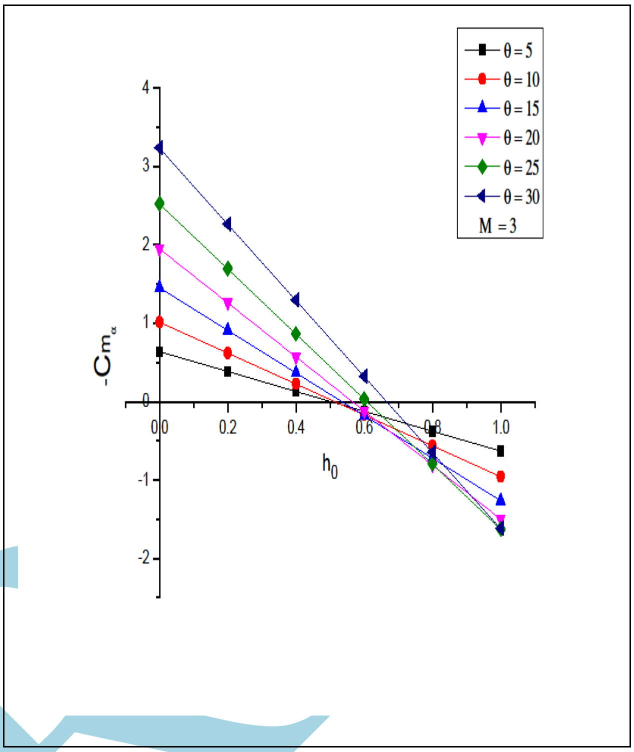


**Fig. 3 Stiffness Derivative Vs pivot position at M=3**

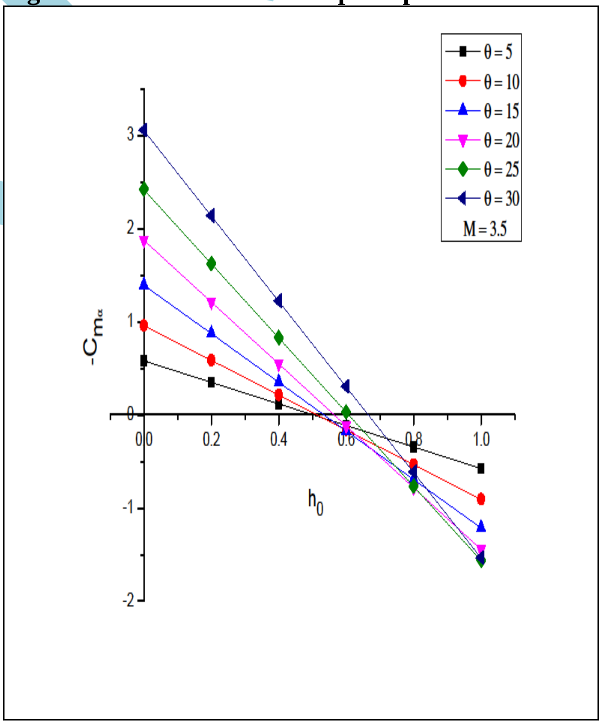


**Fig. 4 Stiffness Derivative Vs pivot position at M=3.5**

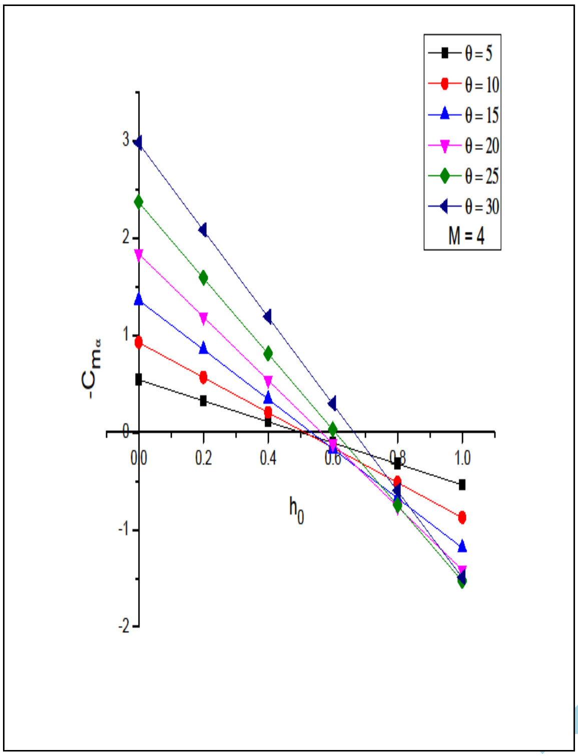


Fig. 5 Stiffness Derivative Vs pivot position at M=4

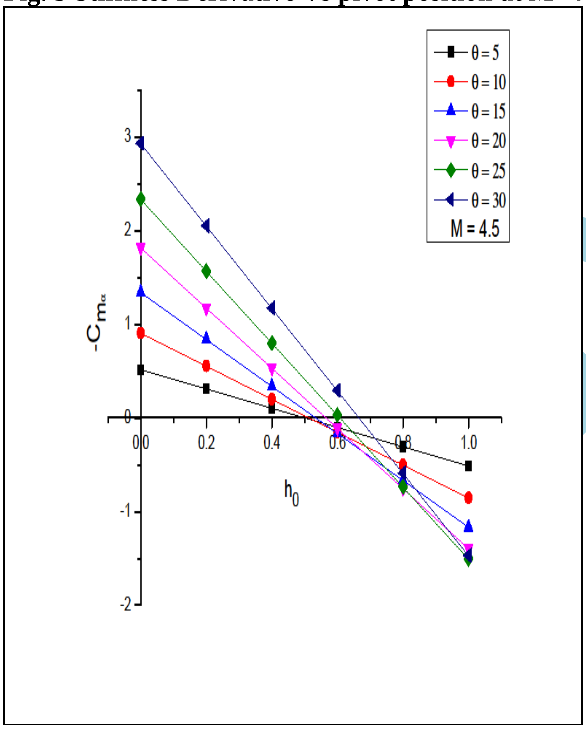


Fig. 6 Stiffness Derivative Vs pivot position at M=4.5

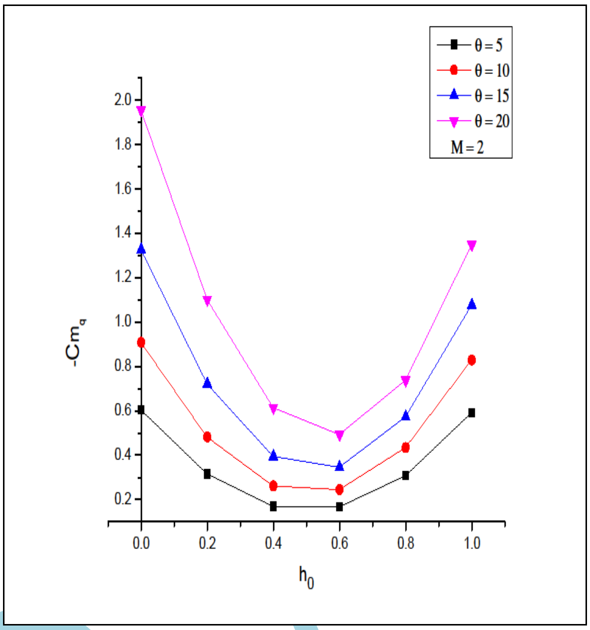


**Fig.7 Damping derivative Vs pivot position at M=2**

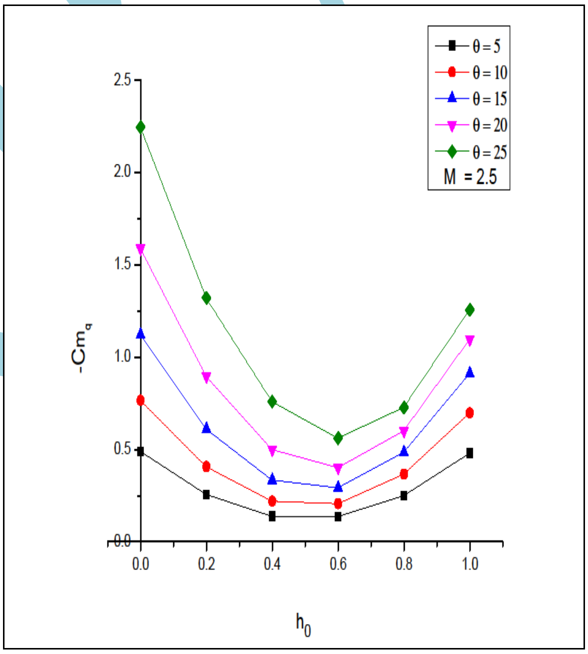


**Fig.8 Damping derivative Vs pivot position at M=2.5**

Figs 7 to 8 show the variation of damping derivatives with pivot positions from 0 to 1 at Mach numbers M = 2 to 2.5 for different semi-vertex angles from $\theta$ = 5 to 20 and from 5 to 25 degrees. From the figure it is seen that initially the damping derivative decreases with the pivot position takes a minimum value then again starts increasing and this trend continues for all the semi-vertex angle. It is also observed that for the lowest value of semi-vertex angle of five degrees due to flat pressure distribution on the wedge surface the center of position is lying between h = 0.4 to 0.6 and this range reduces with the increase in the semi-vertex angle. The maximum value of semi-vertex angle was kept 20 and 25 degrees at Mach 2.0 and 2.5 to satisfy the condition of shock waves being attached with the nose of the wedge.

 

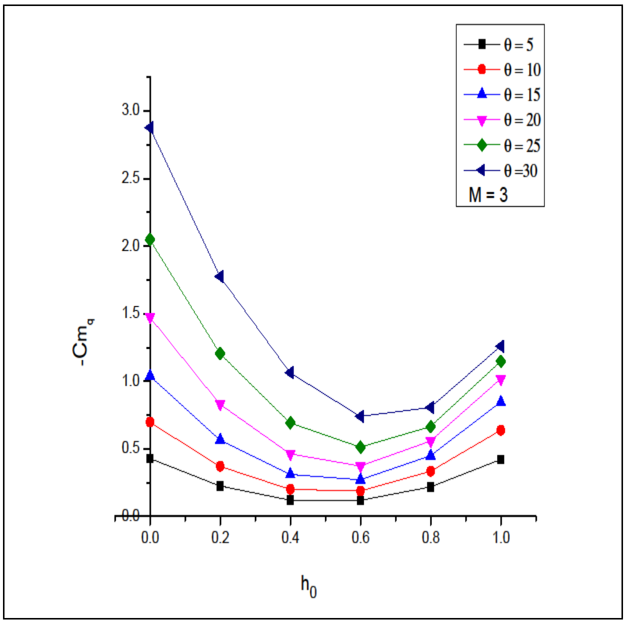


**Fig. 9 Damping derivative Vs pivot position at M=3**

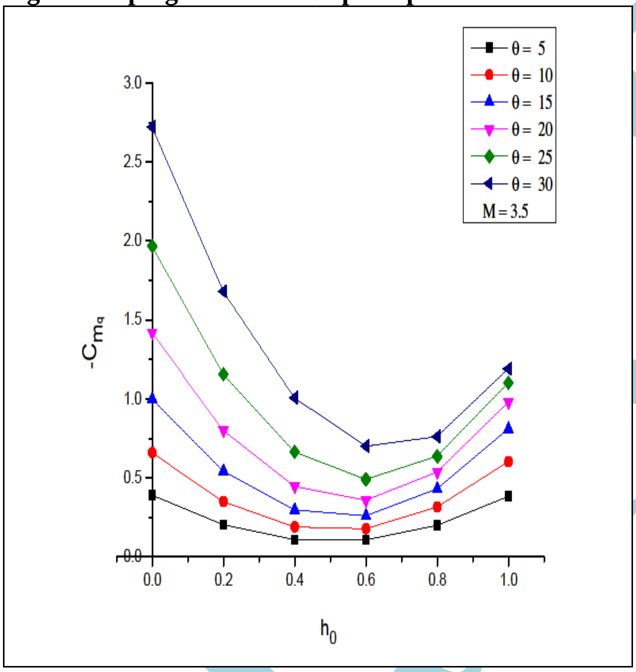


**Fig. 10 Damping derivative Vs pivot position at M=3.5**

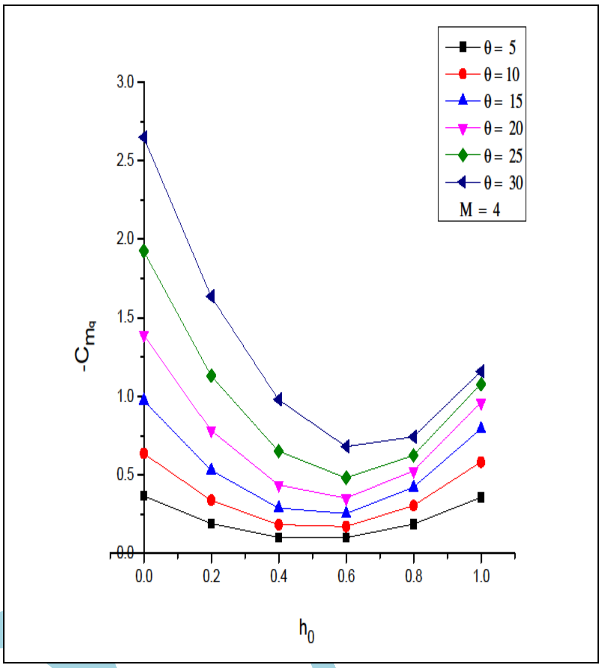


**Fig. 11 Damping derivative Vs pivot position at M=4**

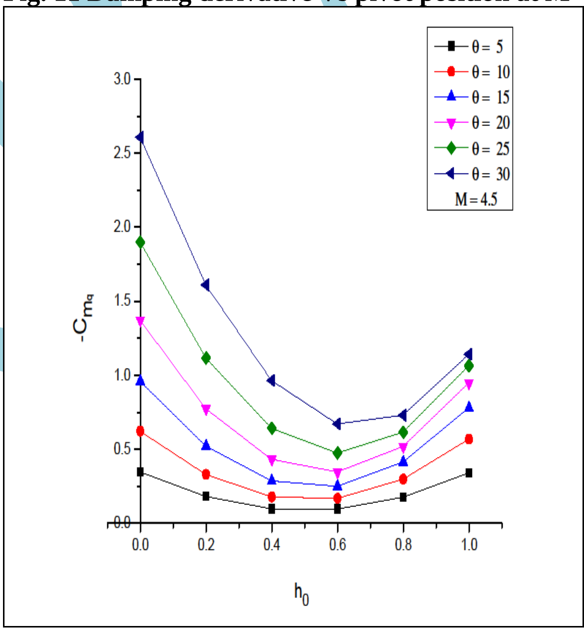


**Fig. 12 Damping derivative Vs pivot position at M=4.5**

Results for Mach 3, 3.5, 4, and 4.5 are presented in Figs 9 to 12. One of the observation from all the figures from 9 to 12 is that the semi-vertex angle 5 degrees is not desirable at these Mach numbers as the shock will be close to the body and the angle between the wedge surface and shock wave will be very small, in view of this surface pressure distribution will be like flat curve which is reflected when the damping derivatives are plotted. As in the case of the stiffness derivative it was seen that stiffness derivative decreases with Mach number as well as the center of pressure is getting shifted towards the trailing edge, here also it is seen that the damping derivatives too decrease with Mach number, however, the center of pressure has got a range depending upon the Mach number, semi-vertex angle and the surface pressure distribution.

## IV. CONCLUSIONS

 

We can observe the following conclusions from above discussion:

- We observe linear decrements in Stiffness derivatives with the increase in the Mach number for all the cases of the present study.
- Damping derivative first decreases then attains a minima and then increases with the increase in pivot position.
- With the increase in the semi-vertex angle the center of pressure is getting shifted towards the trailing edge of the wedge for stiffness derivatives, whereas for damping derivatives the trend is the same initially but later it attains a minima and then again it increase, and the attainment of minima it remains for a range of pivot position and more so this range increases for very low values of the semi-vertex angle. Hence, it is suggested to avoid lowest values of semi-vertex angle for higher supersonic Mach numbers
- Viscous effects, secondary wave reflections, real gas effects and bluntness of leading edge is not considered in the present study.

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