

Computation of Stiffness derivative for an unsteady delta wing with curved leading edges

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Abstract. An attempt has been made to derive the expressions for stiffness derivative for a delta wing with curved leading edges for an unsteady high speed flow. It is evident from the figures that stiffness derivative decreases as Mach number increases. It is also seen that as amplitude of half sine wave increases the stiffness derivative also increases. This is an improvement of previous theory as the effect of leeward surface are included in the present work.

Key words: Unsteady, Hypersonic, Delta wing, Curved leading edge

I. INTRODUCTION

When a flight vehicle experiences a change in both pitch angle and incidence angle simultaneously, the moment derivatives due to pitch rate and that due to the incidence rate have to be evaluated separately to assess the overall stability. In the present work, Appleton method has been combined with Ghosh's unified hypersonic similitude to develop an unsteady piston theory for the prediction of Stiffness derivative of a delta wing with attached Shock. A similitudinal approach is developed by invoking strip theory for quasi steady flow by Crasta and Khan which is now extended to unsteady theory to predict the formulae for stiffness derivative of a delta wing with attached shock case. The real gas effects and effect of viscosity have not been considered.

II. LITERATURE ANALYSIS

(Ghosh,1984) in his theory invokes strip theory which can be applied if $\tan r_1 \ll 1$ or $\tan \bar{\phi} \cdot AR^{-1} \ll 1$ where $\bar{\phi}$ is the shock standoff angle at the leading edge and AR is the aspect ratio. For hypersonic Mach number $\bar{\phi} \ll 1$ so that $\tan \bar{\phi} \approx \bar{\phi}$. According to (Ghosh's ,1986) unified similitude a further restriction is necessary such that $E = \sin^{-1}(1/M_{2x}) - \bar{\phi} \leq 0.3$ where M_{2x} is the axial component of the Mach number behind the bow shock.

Consider a delta wing with arbitrary curved leading edge at large mean angle of incidence α_0 .

The equation to the curved leading edge is

$$Z = x \cot \varepsilon - a_F \sin\left(\frac{2\pi x}{L}\right) - a_H \sin\left(\frac{2\pi x}{L}\right) \quad (1)$$

Where

a_F and a_H are the amplitude of the full and half sine waves and L is the chord length of the wing.

$$\text{Wing area} = L^2 \left(\cot \varepsilon - \frac{4A_H}{\pi} \right) \quad \text{Where } A_H = \frac{a_H}{L} \quad (2)$$

$$AR = b^2/\text{wing area} = \frac{4 \cot^2 \varepsilon}{\left(\cot \varepsilon - \frac{4A_H}{\pi} \right)}$$

At any span-wise location we have,

$$-\left(\frac{\partial \bar{M}}{\partial \theta} \right)_1 = \rho_2 a_2 u_\infty c_1 L_1 \left(\frac{1}{2} - k_1 \right) + \frac{p_\theta \gamma M_\theta^2 L_1^2}{\cos \mu_\theta} \left(\frac{1}{2} - k_1 \right) \quad (3)$$

Using the equations from (1) to (3) we derive the expression for stiffness derivative as

$$-c_{m\dot{\theta}} = \frac{4 \sin \psi c_1}{M \left(\cot \varepsilon - \frac{4A_H}{\pi} \right)} \left\{ \left(\frac{1-k}{3} \right) \cot \varepsilon + \frac{1}{\pi} \left(\frac{A_F}{2} + A_H(2k-1) \right) \right\} + \frac{4}{\left(\cot \varepsilon - \frac{4A_H}{\pi} \right)} \frac{\rho \theta M_\theta^2}{\rho_\infty M_\infty^2} \frac{1}{\sqrt{M_\theta^2 - 1}} \left\{ \left(\frac{1-k}{3} \right) \cot \varepsilon + \frac{1}{\pi} \left(\frac{A_F}{2} + A_H(2k-1) \right) \right\} \quad (4)$$

Where A_H and A_F are defined as

$$A_H = \frac{a_H}{L} = \frac{a_H}{b} 2 \cot \varepsilon$$

$$A_F = \frac{a_F}{L} = \frac{a_F}{b} 2 \cot \varepsilon$$

Based on the above theory results are obtained for various geometrical, and inertia parameters and the same have been discussed.

III. RESULTS AND DISCUSSION

Stiffness derivative variation with pivot position for Mach 5 is pictured in Figure1. There is a linear decrement in Stiffness derivative with the pivot position and increases with increment in amplitude of the half sine wave and decreases with the decrease in amplitude. The reason behind this may be the increase and decrease in the plan form area of the delta wing also due to the change in the shape of the leading edge of the wing. When there is change in the sign of the amplitude the wing leading edge becomes convex to concave.

Stiffness derivative variation with pivot position for Mach 7 is pictured in Figure2. It is observed that stiffness derivative value decreases with increase in Mach number and there is marginal shift in the center of pressure due to the variation in the leading edge shape of the wing otherwise the same trend as above is been observed as seen in Figure.1.

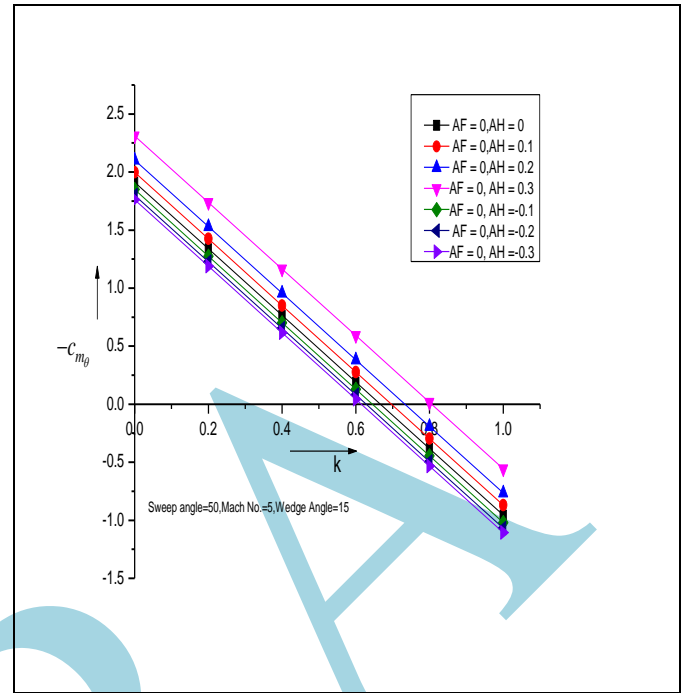


Figure. 1: Stiffness derivative versus pivot position For Mach number 5

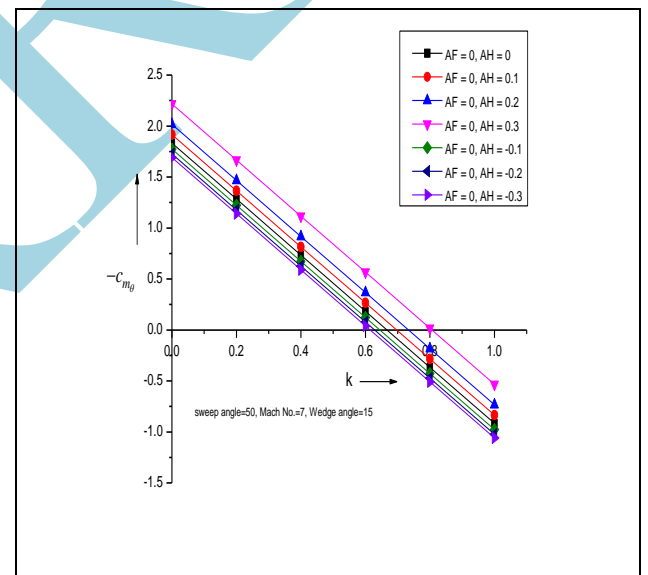


Figure2: Stiffness derivative versus pivot position for Mach number 7

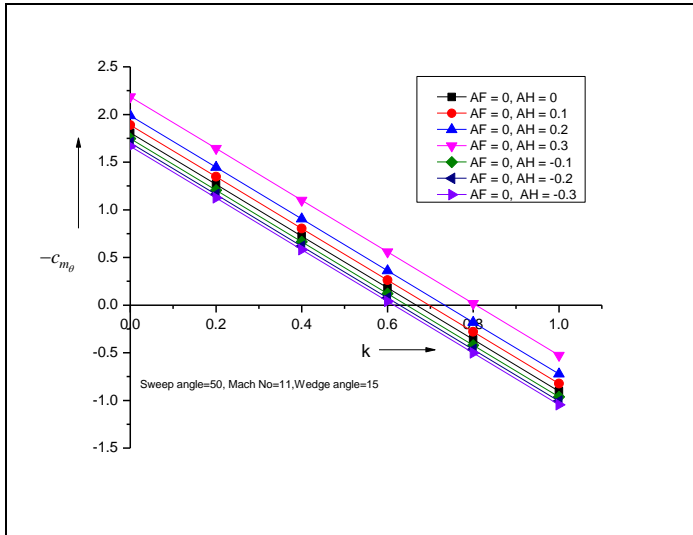


Figure 3: Stiffness derivative versus pivot position for Mach number 11

Stiffness derivative variation with pivot position for Mach 11 is pictured in Figure 3. The value of Stiffness derivative is reduced when Mach number increases from 5 to 11 and the location of the center of pressure remains unchanged.

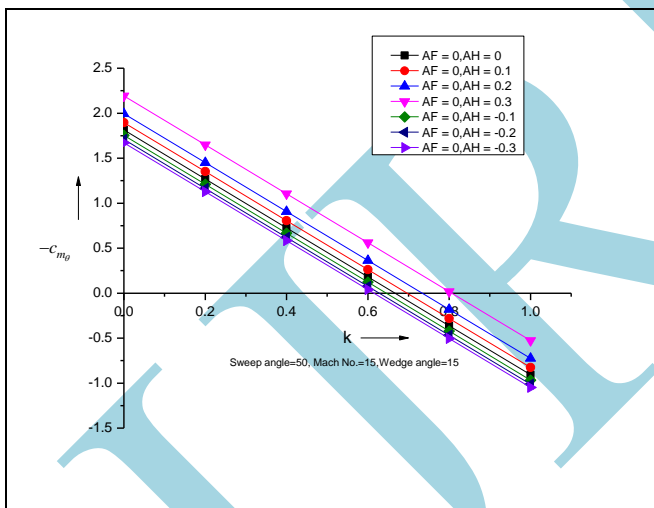


Figure 4: Stiffness derivative versus pivot position for Mach number 15

Stiffness derivative variation with pivot position for Mach 15 is pictured in Figure 4. The value of Stiffness derivative is drastically reduced when Mach number increases from 5 to 15. Otherwise the similar trend as above is seen.

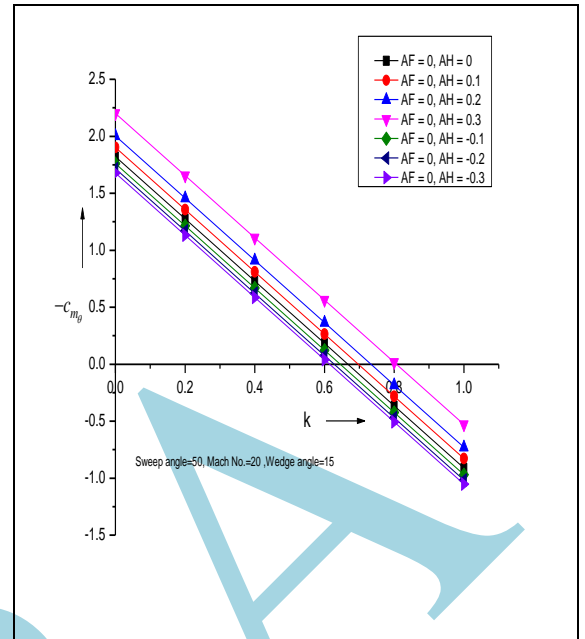


Figure 5: Stiffness derivative versus pivot position for Mach number 20

Stiffness derivative variation with pivot position for Mach 20 is presented in Figure 5. The value of Stiffness derivative is low when Mach number increases from 5 to 20. There is no much difference in the value of stiffness derivative when Mach number increases from 15 to 20 thus exhibiting the Mach number independence principle.

IV. CONCLUSION

From the above discussion we can draw the following conclusions:

1. There is continuous decrease in the stiffness derivatives with increase in Mach number and after certain Mach number it becomes independent of Mach number.
2. The leading edge shape of the wing plays an important role as due to the change in leading edge shape from convex to concave the plan form area will change.
3. A definite pattern is observed that for straight leading edge all the results are taken as the reference results and then convexity and concavity effects are reflected on the either side of the reference results.
4. The wings with curved leading edges will perform better in comparison to the straight leading edge due to the substantial shift in the plan form area of the wing in one case towards the leading edge and the other is towards the trailing edge.

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