Analysis of Thick Isotropic Beam Using Trigonometric Shear Deformation Theory

D. H.Tupe[1], A. G.Dahake[2]

[1] Assistant Professor, Department of Civil Engineering, Deogiri Institute of Engineering And Management Studies, Aurangabad (M.S)-431005, India.

[2] Associate Professor, Department of Civil Engineering, Marathwada Institute of Technology
Aurangabad (M.S)-431005, India.
*Corrosponding Author: durgeshtupe@gmail.com

Abstract: **In the present study, a trigonometric shear deformation theory is developed for static flexural analysis of thick isotropic beams. The number of variables in the present theory is same as that in the first order shear deformation theory. In this theory the the sinusoidal function is used in displacement field in terms of thickness coordinate to represent the shear deformation effect and satisfy the zero transverse shear stress condition at top and bottom surface of the beams. The Governing differential equation and boundary conditions of the theory are obtained by using Principle of virtual work. The fixed isotropic beam subjected to varying load is examined using present theory. The numerical results have been computed for various lengths to thickness ratios of the beams and the results obtained are compared with those of Elementary, Timoshenko, trigonometric and other higher order refined theories and with the available solution in the literature.**

Keywords: **Thick beam, shear deformation, isotropic beam, transverse shear stress, static flexure, trigonometric shear deformation theory, principle of virtual work**

1.Introduction

It is well-known that elementary theory of bending of beam based on Euler-Bernoulli hypothesis disregards the effects of the shear deformation and stress con- centration. The theory is suitable for slender beams and is not suitable for thick or deep beams since it is based on the assumption that the transverse normal to neutral axis remains so during bending and after bending, implying that the transverse shear strain is zero. Since theory neglects the transverse shear deformation, it underestimates deflections in case of thick beams where shear deformation effects are significant.

Bresse [1], Rayleigh [2] and Timoshenko [3] were the pioneer investigators to include refined effects such as rotatory inertia and shear deformation in the beam theory. Timoshenko showed that the effect of transverse vibration of prismatic bars. This theory is now widely referred to as Timoshenko beam theory or first order shear deformation theory (FSDT) in the literature. In this theory trans- verse shear strain distribution is assumed to be constant through the beam thickness and thus requires shear correction factor to appropriately represent the strain energy of deformation. Cowper [4] has given refined expression for the shear correction factor for different crosssections of beam. The accuracy of Timoshenko beam theory for transverse vibrations of simply supported beam in respect of the funda- mental frequency is verified by Cowper [5] with a plane stress exact elasticity solution. To remove the discrepancies in classical and first order shear defor- mation theories, higher order or refined shear defor- mation theories were developed and are available in the open literature for static and vibration analysis of beam.

Levinson [6], Bickford [7], Rehfield and Murty [8], Krishna Murty [9], Baluch, Azad and Khidir [10], Bhimaraddi and Chandrashekhara [11] presented parabolic shear deformation theories assuming a higher variation of axial displacement in terms of thickness coordinate. These theories satisfy shear stress free boundary conditions on top and bottom surfaces of beam and thus obviate the need of shear correction factor.

There is another class of refined theories, which includes trigonometric functions to represent the shear deformation effects through the thickness. Vlasov and Leontev [12], Stein [13] developed refined shear deformation theories for thick beams includ- ing sinusoidal function in terms of thickness coordi- nate in displacement field. However, with these the- ories shear stress free boundary conditions are not satisfied at top and bottom surfaces of the beam. Further Ghugal and Dahake [14] developed a trigonometric shear deformation theory for flexure of thick beam or deep beams taking into account transverse shear deformation effect. The number of variables in the present theory is same as that in the first order shear deformation theory. The trignometric function is used in displacement field in terms of thickness co- ordinate to represent the shear deformation effects. A study of literature by Ghugal and Shimpi [20] indicates that the research work dealing with flexural analysis of thick beams using refined trigonometric and hyperbolic shear deformation theories is very scarce and is still in infancy. In this paper development of trigonometric theory and its application to thick cantilever beams is presented.

2.Formulation of Problem

Consider a thick isotropic fixed beam of length L in x direction, Width b in y direction and depth h as shown in Figure 1. Where x, y, z are Cartesian coordinates. The beam is subjected to transverse load of intensity q (x) per unit length of beam. Under displacement, Axial bending stress and transverse shear stress are required to be determined.

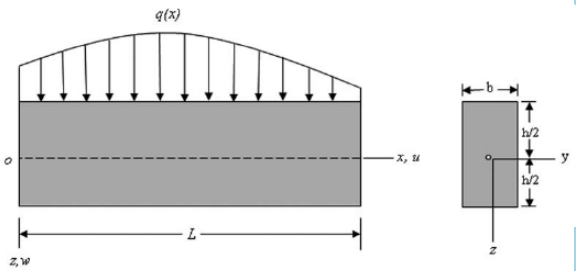

Figure 1: Simply supported beam bending under x-z plane

2.3 The Displacement field:

The displacement field of the present beam theory can be expressed as follows. The trigonometric function is assigned according to the shearing stress distribution through the thickness of beam.

$$u(x,z) = -z\frac{dw}{dx} + \frac{h}{\pi}\sin\frac{\pi z}{h}\phi(x) \qquad (1)$$

$$w(x,z) = w(x) \qquad (2)$$

Where

u = Axial displacement in x direction which is func-

tion of x and z.

w = Transverse displacement in z direction which is function of x.

φ = Rotation of cross section of beam at neutral axis due to shear which is an unknown function to be de- termined and it is a function of x.

Normal strain:

$$\varepsilon_x = \frac{\partial u}{\partial x} = -z\frac{d^2w}{dx^2} + \frac{h}{\pi}\sin\frac{\pi z}{h}\frac{d\phi}{dx} \qquad (3)$$

Shear strain:

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \cos\frac{\pi z}{h}\phi \qquad (4)$$

Stresses:

The one dimensional Hookes law is applied for isotropic material, stress x is related to strain x and shear stress is related to shear strain by the following constitutive relations.

$$\sigma_x = E\varepsilon_x = -zE\frac{d^2w}{dx^2} + E\frac{h}{\pi}\sin\frac{\pi z}{h}\frac{d\phi}{dx} \qquad (5)$$

$$\tau_{xz} = G\gamma_{xz} = G\cos\frac{\pi z}{h}\phi \qquad (6)$$

where E and G are the elastic constants of the beam material.

2.4 Governing differential equations:

Governing differential equations and boundary conditions are obtained from Principle of virtual work. Using equations for stresses, strains and principle of virtual work, variationally consistent differential equations for beam under consideration are obtained. The principle of virtual work when applied to beam leads to:

$$b\int_{x=o}^{x=L}\int_{z=-h/3}^{z=h/3}\left(\sigma_x\delta\varepsilon_x + \tau_{xz}\delta\gamma_{xz}\right)dxdz - \int_{x=0}^{x=L} q\delta w dx = 0 \qquad (7)$$

Where δ = variational operator.

Employing Greens theorem in above Equation successively, we obtain the coupled Euler-Lagrange equations which are the governing differential equations and associated boundary conditions of the beam. The governing differential equations obtained are as follows:

$$EI\left[\frac{8}{27}\frac{d^4w}{dx^4} - A_0\frac{d^3\phi}{dx^3}\right] = q(x) \qquad (8)$$

$$EI\left[A_0\frac{d^3w}{dx^3} - B_0\frac{d^2\phi}{dx^2}\right] + GAC_0\phi = 0 \qquad (9)$$

Where A_0 , B_0 and C_0 are the stiffness coefficients in governing equations. The associated consistent natural boundary form along the edges x = 0 and x = L.

$$V_x = EI\left[\frac{8}{27}\frac{d^3w}{dx^3} - A_0\frac{d^2\phi}{dx^2}\right] = 0 \qquad (10)$$

Where w is Prescribed.

$$M_x = EI\left[\frac{8}{27}\frac{d^2w}{dx^2} - A_0\frac{d\phi}{dx}\right] = 0 \qquad (11)$$

Where w is Prescribed.

$$M_a = EI\left[A_0\frac{d^2w}{dx^2} - B_0\frac{d\phi}{dx}\right] = 0 \qquad (12)$$

Where φ is Prescribed.
The flexural behaviour of beam is given by solution of above equations 8 and 9 by discarding all terms containing time derivatives and satisfying the asso- ciate boundary conditions. The stiffness coefficient used in governing equations Equations 8, 9, 10,11 and 12 are described as below:

$$A_0 = \left[\frac{24}{\pi^3}\sin\left(\frac{\pi}{3}\right) - \frac{8}{\pi^2}\cos\left(\frac{\pi}{3}\right)\right] \qquad (13)$$

$$B_0 = \left[\frac{4}{\pi^2} - \frac{6}{\pi^3}\sin\left(\frac{2\pi}{3}\right)\right] \qquad (14)$$

$$C_0 = \left[\frac{1}{3} - \frac{1}{2\pi}\sin\left(\frac{2\pi}{3}\right)\right] \qquad (15)$$

2.5 The general solution of governing equilib- rium equations of the Beam

The general solution for transverse displacement

w(x) and φ(x) can be obtained from equation 8 and

9 by discarding the terms containing time (t) deriva- tives. Integrating and rearranging the equation 8, we obtained the following equation

$$\frac{d^3w}{dx^3} = A_0\frac{27}{8}\frac{d^2\phi}{dx^2} + \frac{27}{8}\frac{Q(x)}{D} \qquad (16)$$

Where, Q(x) is generalised shear force for beam.

$$Q(x) = \int_0^x q\,dx + k1 \qquad (17)$$

The second governing equation 9 can be written as:

$$\frac{d^3w}{dx^3} = \frac{B_0}{A_0}\frac{d^2\phi}{dx^2} - \beta\phi \qquad (18)$$

Now using equations 16 and 18 a single equation in

terms of φ is obtained as:

$$\frac{d^2\phi}{dx^2} - \lambda^2\phi = \frac{Q(x)}{\alpha D} \qquad (19)$$

The general solution of equation 19 is as follows:

$$\phi = k_2\cosh\lambda x + k_3\sinh\lambda x - \frac{Q(x)}{\beta D} \qquad (20)$$

$$\alpha = \left(\frac{B_0}{A_0} - \frac{27}{8}A_0\right) \qquad \beta = \left(\frac{GAC_0}{DA_0}\right)$$

$$\lambda^2 = \frac{\beta}{\alpha} \qquad D = EI$$

The equation of transverse displacement w(x) is obtained by substituting the expression of φ(x) in equation 18 and integrating it thrice with respect to x. The general solution for w(x) is obtained as follows:

$$EIw(x) = \iiiint q\,dx\,dx\,dx\,dx + \frac{D}{\lambda^3}\left(\frac{B_0}{A_0}\lambda^2 - \beta\right)$$
$$\left(k_2\sinh\lambda x + k_3\cosh\lambda x\right) + \frac{k_1x^3}{6} + k_4\frac{x^2}{2}$$
$$+k_5x + k_6 \qquad (21)$$

where k1, k2, k3, k4, k5 and k6 are the constants

of integration and can be obtained by applying the

boundary conditions of the beams.

3. Illustrative example:

In order to prove the efficiency of the present theory, the following numerical examples are considered.
The following material properties for beam are used.

Material properties:

1. Modulus of Elasticity E=210GPa
2. Poissions ratio = 0.30

3.Density=7800 Kg/m^3

3.1 Example 1: Cantilever beam with uniformly varying load q(x) = q0 (x/L)

A Cantilever beam has its origin at left hand side support and is fixed at x = 0 and free at x=L. The beam is subjected to uniformly varying load q(x)

$$q(x) = q_0\left(\frac{x}{L}\right)$$

The boundary conditions associated with this problem are as follows.

At fixed end:

$$EI\frac{d^3w}{dx^3} = 0 \qquad \text{at} \qquad x = L \qquad (22)$$

$$.EI\frac{d^2w}{dx^2} = 0 \qquad \text{at} \qquad x = L \qquad (23)$$

$$EI\frac{d^2\phi}{dx^2} = 0 \qquad \text{at} \qquad x = L \qquad (24)$$

$$EI\frac{d\phi}{dx} = 0 \qquad \text{at} \qquad x = L \qquad (25)$$

At free end:

$$EI\frac{dw}{dx} = 0 \qquad \text{at} \qquad x = 0 \qquad (26)$$

$$EI\phi = 0 \qquad \text{at} \qquad x = 0 \qquad (27)$$

$$EIw = 0 \qquad \text{at} \qquad x = L \qquad (28)$$

General expressions obtained for w(x) and φ(x) are as follows

$$w(x) = \frac{27}{8}\left(\frac{x^5}{L^5} - 10\frac{x^3}{L^3} + 20\frac{x^2}{L^2}\right) - 10\frac{27}{8}\frac{B_0}{C_0}\frac{E}{G}\frac{h^2}{L^2}\left(\frac{1}{6}\frac{x^3}{L^3} - \frac{1}{2}\frac{x^2}{L^2}\right) + 5\frac{27}{8}\frac{27}{8}\frac{A_0^2}{C_0}\frac{E}{G}\frac{h^2}{L^2}\,\text{x}\left(\frac{\cosh\lambda x - \sinh\lambda x - 1}{\lambda L} + \frac{x}{L}\right) \qquad (27)$$

$$\phi(x) = \frac{1}{2}\frac{27}{8}\frac{A_0}{C_0}\frac{q_0 L}{Gbh}\left(-\cosh\lambda x + \sinh\lambda x + 1 - \frac{x^2}{L^2}\right) \qquad (28)$$

The axial displacement, stresses and transverse shear stress obtained based on above solutions are as follows:

$$\bar{u} = -\frac{z}{h}\frac{1}{10}\frac{L^3}{h^3}\frac{27}{8}\left(5\frac{x^4}{L^4} - 30\frac{x^2}{L^2} + 40\frac{x}{L}\right) - 5\frac{z}{h}\frac{E}{G}\frac{L}{h}\frac{A_0^2}{C_0}\frac{729}{64}\left(-\cosh\lambda x + \sinh\lambda x + 1\right)\times\frac{z}{h}\frac{27}{8}\frac{B_0}{C_0}\frac{E}{G}\frac{h}{L}\left(\frac{x^2}{2L^2} - \frac{x}{L}\right) + \frac{A_0}{C_0}\frac{E}{G}\frac{L}{h}\frac{27}{16}\times\frac{1}{\pi}\sin\frac{\pi z}{h}\times\left(-\cosh\lambda x + \sinh\lambda x + 1 + \frac{x^2}{L^2}\right) \qquad (29)$$

$$\bar{\sigma}x = \left[\frac{27}{8}\left(20\frac{x^3}{L^3} - 60\frac{x}{L} + 40\right) - \frac{270}{8}\frac{E}{G}\frac{h^2}{L^2}\frac{B_0}{C_0}\left(\frac{x}{L} - 1\right)\right] \times\left(-\frac{z}{h}\frac{1}{10}\frac{L^2}{h^2}\right) - \left[\frac{z}{h}\frac{A_0^2}{C_0}\frac{729}{128}\frac{E}{G}\left(\lambda L\cosh\lambda x - \lambda L\sinh\lambda x\right)\right] \left[\frac{A_0}{C_0}\frac{E}{G}\frac{27}{16}\frac{1}{\pi}\sin\frac{\pi z}{h}\left(\lambda L\cosh\lambda x - \lambda L\sinh\lambda x - \frac{2x}{L}\right)\right] \qquad (30)$$

$$\bar{\tau}_{zx}^{CR} = \frac{A_0}{C_0}\frac{L}{h}\frac{27}{16}\cos\frac{\pi z}{h}\left(\sinh\lambda x - \cosh\lambda x + 1 - \frac{x^2}{L^2}\right) \qquad (31)$$

4.Numerical Results:

The numerical results for axial displacement, transverse displacement, bending stress and transverse shear stress are presented in following non dimensional form and the values are presented in Table 1 and Table 2

$$\bar{w} = \frac{10Ebh^3}{q_0 L^4}w \qquad \bar{u} = \frac{Eb}{q_0 L^4}u$$

$$\bar{\sigma}_x = \frac{b\sigma_x}{q_0} \qquad \bar{\tau}_{zx} = \frac{b\tau_{zx}}{q_0}$$

Table 1: Non-Dimensional axial displacement $\bar{u}$ at (x=L/2 , z=h/3), Transverse deflection $\bar{w}$ at (x=L, z=0), axial stress $\bar{\sigma}_x$ at (x=0, z=h/3), maximum transverse shear stress $\bar{\tau}_{ZX}^{CR}$ and $\bar{\tau}_{ZX}^{EE}$ at (x=0.01L , z=0), of simply supported beam subjected to Varying load for Aspect Ratio 4.

Source	**Theory**	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{ZX}^{CR}$	$\bar{\tau}_{ZX}^{EE}$

Present	TSDT	39.8826	-113.8951	-86.8530	-3.4537	43.8046
Ghugal and Sharma	HPSDT	39.8451	-113.8138	-82.3315	-3.1320	41.7179
Krishna Murty	HSDT	39.8453	-113.8134	-86.9805	-3.4677	37.1810
Timoshenko	FSDT	38.4413	-108.00	-72.00	-3.2982	10.1250
Bernoulli-Euler	ETB	37.1250	-108.00	-72.00	–	10.1250

Table 2: Non-Dimensional axial displacement $\bar{u}$ at (x=L/2 , z=h/3), Transverse deflection $\bar{w}$ at (x=L, z=0), axial stress $\bar{\sigma}_x$ at (x=0, z=h/3), maximum transverse shear stress $\bar{\tau}_{ZX}^{CR}$ and $\bar{\tau}_{ZX}^{EE}$ at (x=0.01L , z=0), of simply supported beam subjected to Varying load for Aspect Ratio 4.

Source	**Theory**	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{ZX}^{CR}$	$\bar{\tau}_{ZX}^{EE}$
Present	TSDT	37.5667	-1702.20	-484.9191	-9.4688	27.5451
Ghugal and Sharma	HPSDT	37.5609	-1702.00	-473.66	-9.1799	30.1377
Krishna Murty	HSDT	37.5667	-1701.87	-485.26	-9.1863	26.1853
Timoshenko	FSDT	38.4412	-1687.33	-449.950	-9.1728	25.3125
Bernoulli-Euler	ETB	37.1250	-1687.50	-450.00	–	25.3125

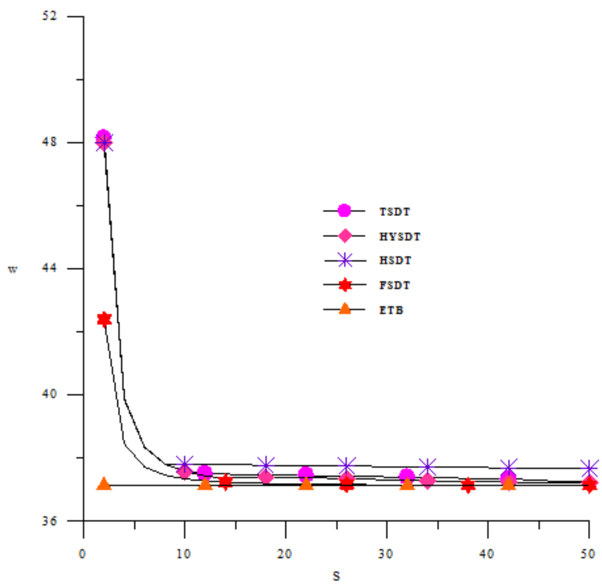

Figure 2: Variation of Transverse Displacement w

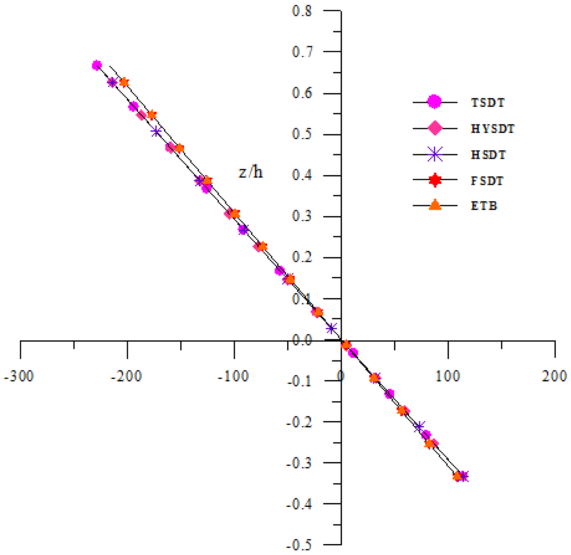

Figure 3: Variation of Maximum Axial displacement (ū) for AS 04

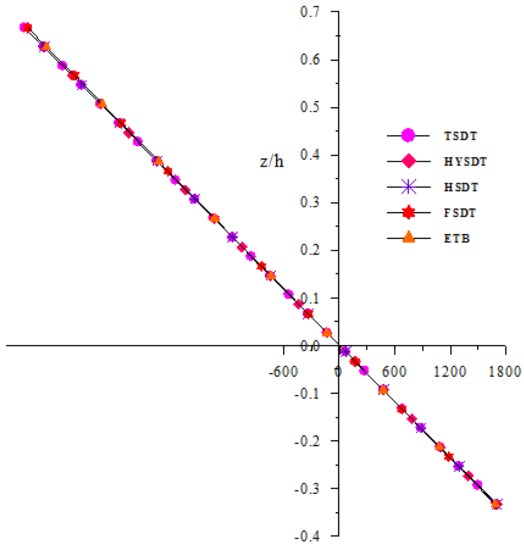

Figure 4: Variation of Maximum Axial displacement (u) for AS 10

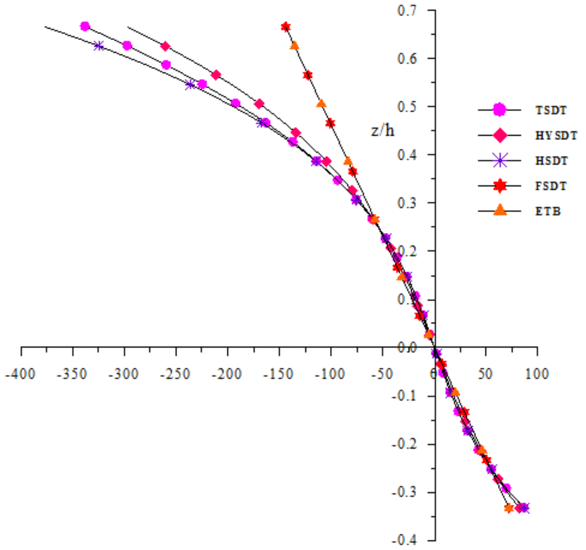

Figure 5: Variation of Maximum Axial stress (σ_x) for AS 04

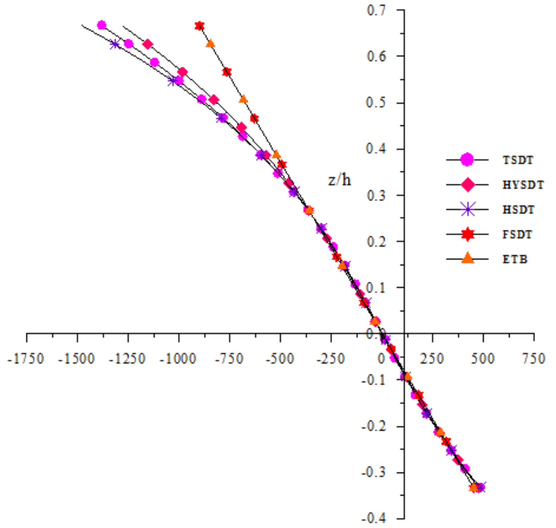

Figure 6: Variation of Maximum Axial stress (σ_x) for AS 10

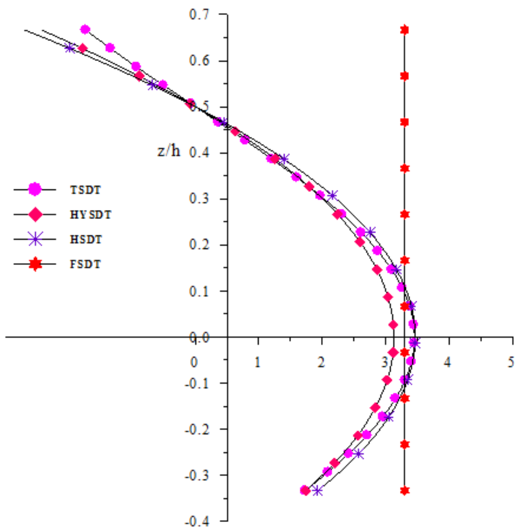

Figure 7: Variation of Transverse shear stress ($\bar{\tau}_{zx}^{CR}$) for AS 04

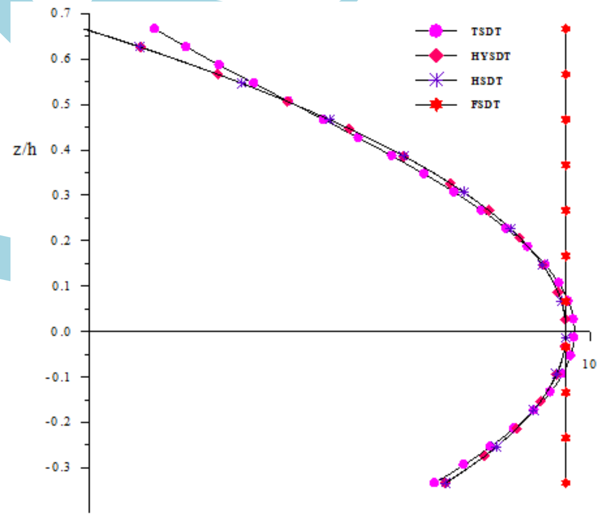

Figure 8: Variation of Transverse shear stress ($\bar{\tau}_{zx}^{CR}$) for AS 10

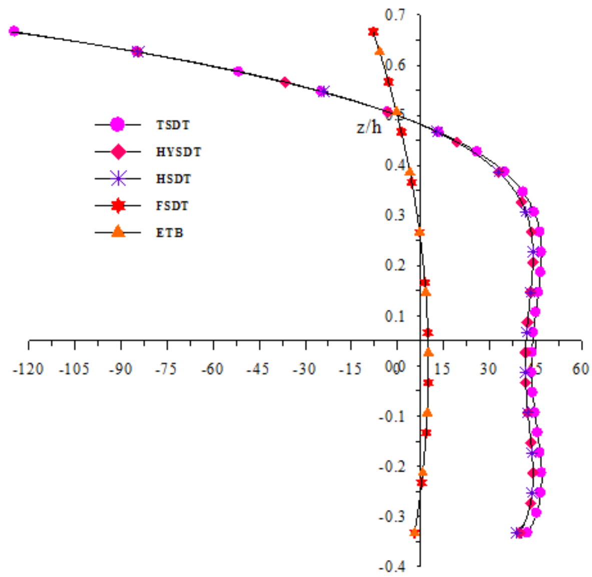

Figure 9: Variation of Transverse shear stress (τ_{zx}^{EE})

for AS 04

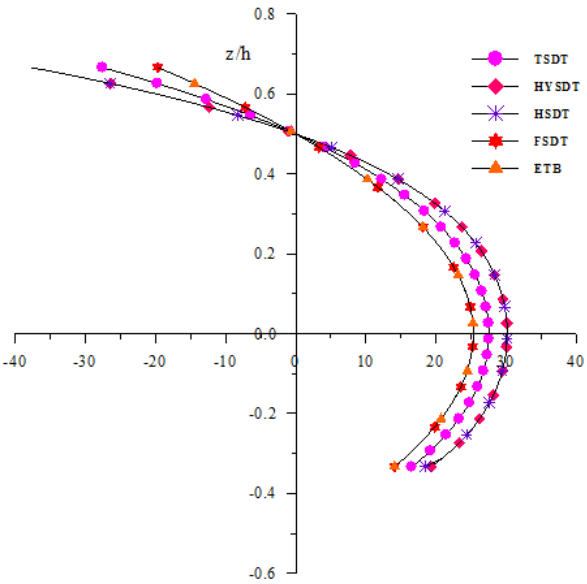

Figure 10: Variation of Transverse shear stress (τ^{EE})

for AS 10

5. Conclusion

From the static flexural analysis of cantilever beam following conclusion are drawn:

1. The result of maximum transverse displacement w obtained by present theory is in excel- lent agreement with those of other equivalent refined and higher order theories. The variation of w for AS 4 and 10 are presented as shown in Figure 2.

2. From Figure 3 and Figure 4, it can be ob- served that, the result of axial displacement u for beam subjected to varying load varies linearly through the thickness of beam for AS 4 and 10 respectively.

3. The variation of maximum non dimensional axial stresses σ_x for AS 4 and 10 of beam as shown in Figure 5 and Figure 6 respectively.

4. The maximum transverse shear stress obtained by present theory using constitutive relation is in good agreement with that of higher order theories for aspect ratio 4 and for aspect ratio 10. The through thickness variation of this stress obtained via constitutive relation are presented graphically in Figure 7 and 8 and those obtained via equilibrium equation are presented in Figures 9 and 10.

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