Thermal Flexural Analysis of Isotropic Plate Using New Trigonometric Shear Deformation Theory

[1]Bausaheb.R. Sontakke, [2]Rushikesh V. Pakhale, [3]Sachin B. Salve

[1] Associate Professor & Head, Department of Mathematics, PratishthanMahavidyalaya, Paithan, (M.S)-431107, India.

[2] Assistant Professor, Department of Mathematics, Deogiri Institute of Engineering and Management Studies, Aurangabad (M.S)-431005, India.

[3] Assistant Professor, Department of Civil Engineering, Deogiri Institute of Engineering and Management Studies, Aurangabad (M.S)-431005, India. ,

[1]brsontakke@rediffmail.com, [2]rvpakhale@gmail.com,[3]ssalve47@ gmail.com,

***Abstract*— In the present study, thermal deformations and transverse shear deformation effects of isotropic plate were analyzed using Trigonometric shear deformation Theory. Analytical formulations and solutions for the thermal stress analysis of simply supported isotropic plate subjected to linear thermal load based on trigonometric deformation are presented. Simply supported isotropic plate is analysed for the axial displacement, transverse displacement, axial bending stress and transverse shear stress. In New Trigonometric Shear Deformation Theory having three variables for the displacement field. The displacement field for analysis of plate is trigonometric. Boundary conditions and governing differential equations of the theory are obtained using the principle of virtual work. The important feature of the theory is that the transverse shear stresses can be obtained directly from the use of constitutive relations, satisfying the stress free boundary conditions at top and bottom surfaces of the plate. Hence, the theory eliminates the need of shear correction factor. Plates with different aspect ratio are studied. Results obtained from isotropic plates subjected to linear thermal load are compared with other shear deformation theory to check the accuracy of the present theory.**

***Index Terms*—Displacement Field, Isotropic beam, Simply Supported plate, Thermal Load**

1.1 Introduction

In this paper, a variationally consistent trigonometric shear deformation theory for thermal stresses is developed. The governing differential equations and boundary conditions are obtained using principle of virtual work. The stiffness matrix is used to find out field variables w, ϕ and ψ by using governing differential equations. The theory is applied to simply supported and uniform isotropic solid plate for thermal stress analyses. A closed-form and general solutions are obtained. The general solutions for field variables w, ϕ and ψ are obtained for plate under consideration using appropriate boundary conditions. The general expressions for displacements and stresses are presented. Results obtained are comparing with those of elementary beam theory, higher order shear deformation theory. The credibility of the present theory is established by accurate evaluation of displacements and thermal stresses.

Theoretical Formulation - The theoretical formulation of a uniform plate based on certain kinematical and physical assumptions is presented. The variationally correct forms of differential equations and boundary conditions, based on assumed displacement field will be obtained using dynamic version of the principle of virtual work.

1.2The plate under consideration- The variationally correct forms of differential equations and boundary conditions, based on the assumed displacement field are obtained using the principle of virtual work. The plate under consideration occupies the following region. Consider a thick isotropic simply supported plate of length a in x direction, Width b in y direction and depth h as shown in Figure 1. Where x, y and z are Cartesian coordinates. The plate is subjected to sinusoidalthermal load of intensity $T(x)$ on whole length of plate. Underthis condition, the axial displacement, transversedisplacement, axial bending stress and transverseshear stress are required to be determined. The beam is made up ofhomogeneous, linearly elastic isotropic material with the principal material axes parallelto the $x$and$y$ axes in the plane of plate. The plate's material obeys the generalized Hook's law.

$$0 \le x \le a; \quad 0 \le y \le b; \quad -h/2 \le z \le h/2 \qquad (1)$$

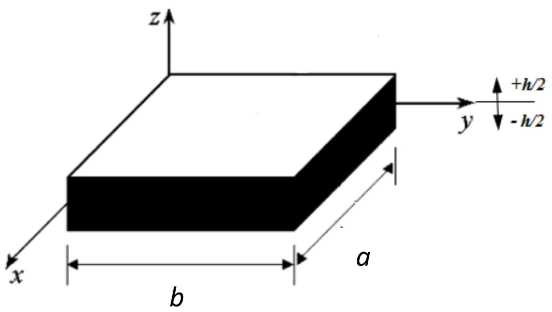

Figure 1: Plate under bending in x-z plane

1.2.2 Assumptions made in the theoretical formulation

1) The displacements are small and therefore strains involved are infinitesimal.

2) The in-plane displacement u in x direction as well as displacement v in y direction consists of three parts:

a. Displacement component analogous to the displacement in classical plate theory of bending.
b. Displacement component due to shear deformation, which is assumed to be trigonometric in nature with respect to thickness coordinate, such that the maximum shear stress occurs at neutral axis.
c. The displacements are small compared to plate thickness.

3) The transverse displacement *w* in *z* direction is assumed to be a function of *x* and *y* coordinates only.

4) The body forces are ignored in the analysis.

5) The plate is subjected to thermal load only.

1.2.1The displacement field of TSDT

Based on the above mentioned assumptions, the displacement field of the present plate theory can be expressed as follows. The trigonometric function is assigned according to theshearing stress distribution through the thickness of plate.

$$u(x,y,z) = -z\frac{\partial w(x,y)}{\partial x} + \left[z - \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)\right]\phi(x,y), \quad (3.2)$$

$$v(x,y,z) = -z\frac{\partial w(x,y)}{\partial x} + \left[z - \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)\right]\psi(x,y), \quad (3.3)$$

$$w(x,y,z) = w(x,y,z)$$

Where*u*and*v*are the in-plane displacement components in the *x* and*y* directionsrespectively, and*w*is the transverse displacement in the *z* direction.

The trigonometricfunction in terms of the thickness coordinate in both the in-plane displacements *u*and*v*is associated with the transverse shear stress distribution through the thickness ofplate. The functions $\phi(x, y)$ and$\psi(x, y)$ are the unknown functions associated with theshear slopes. Normal and shear strains are obtained within the framework of the lineartheory of elasticity using the displacement field.

1.2.1 Strain-displacement relationships

Normal and shear strains are obtained within the framework of linear theory of elasticity using the displacement field given:

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{bmatrix}\frac{\partial}{\partial x} & 0\\ 0 & \frac{\partial}{\partial y}\\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x}\end{bmatrix}\begin{Bmatrix}u\\ v\end{Bmatrix} \text{ and } \begin{Bmatrix}\gamma_{zx}\\ \gamma_{yz}\end{Bmatrix} = \begin{bmatrix}\frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x}\\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y}\end{bmatrix}\begin{Bmatrix}u\\ v\\ w\end{Bmatrix} \quad (4)$$

Substituting the in-plane and transverse displacement field from Eq. (2)and Eq. (3) into Eq. (4), we get

Normal Strain:

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = z\begin{Bmatrix}k_x^c\\ k_y^c\\ k_{xy}^c\end{Bmatrix} + f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}\begin{Bmatrix}u\\ V\end{Bmatrix} \text{ and } \begin{Bmatrix}\gamma_{zx}\\ \gamma_{yz}\end{Bmatrix} = g(z)\begin{Bmatrix}\phi\\ \psi\end{Bmatrix}$$

$$k_x^c = -\frac{\partial^2 w}{\partial x^2},\ k_y^c = -\frac{\partial^2 w}{\partial y^2},\ k_{xy}^c = -\frac{\partial^2 w}{\partial x \partial y},$$

$$k_x^s = \frac{\partial \phi}{\partial x},\ k_y^s = \frac{\partial \psi}{\partial y},\ k_{xy}^s = \frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x} \quad (5)$$

$$u(x,y,z) = -z\frac{\partial w(x,y)}{\partial x} + \left[z - \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)\right]\phi(x,y),$$

$$v(x,y,z) = -z\frac{\partial w(x,y)}{\partial x} + \left[z - \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)\right]\psi(x,y), \quad (6)$$

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{21} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x - \alpha_x T\\ \varepsilon_y - \alpha_y T\\ \gamma_{xy}\end{Bmatrix} \text{ and}$$

$$\begin{Bmatrix}\tau_{zx}\\ \tau_{yz}\end{Bmatrix} = \begin{bmatrix}Q_{44} & 0\\ 0 & Q_{55}\end{bmatrix}\begin{Bmatrix}\gamma_{zx}\\ \gamma_{yz}\end{Bmatrix} \quad (7)$$

1.2.5 Stress-strain relationships

For a plate of constant thickness, composed of isotropic material, one dimensional Hooke's law is applied for isotropic material, stress(σ_x) is related to strain(ε_x)and shear stress is related to shear strain by the following constitutive relations: The $(\sigma_x, \sigma_y, \tau_{xy}, \tau_{zy}, \tau_{zx})$ are the stress components, $(\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{zy}, \gamma_{zx})$are the strain components, *E* is the modulus of elasticity, $\alpha_x$and$\alpha_y$are the co efficient of

thermal expansion along $x$and$y$ directions respectively and$T$ is the thermal load.
By inserting Eq. (3.5) in to Eq. (3.7) we get:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = z\begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{21} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}k_x^c\\ k_y^c\\ k_{xy}^c\end{Bmatrix} + f(z)\begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{21} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix} - z\begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{21} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}\alpha_x T\\ \alpha_y T\\ 0\end{Bmatrix}$$

and

$$\begin{Bmatrix}\tau_{zx}\\ \tau_{yz}\end{Bmatrix} = g(z)\begin{bmatrix}Q_{44} & 0\\ 0 & Q_{55}\end{bmatrix}\begin{Bmatrix}\phi\\ \psi\end{Bmatrix} \tag{8}$$

Where $Q_{ij}$are the plane stress reduced elastic constants in the material axes of theplate defined as:

$$Q_{11} = \frac{E}{1-\mu^2},\ Q_{12} = \frac{\mu E}{1-\mu^2},\ Q_{22} = \frac{E}{1-\mu^2},$$

$$Q_{66} = G_{12},\ Q_{55} = G_{13},\ Q_{44} = G_{23} \tag{9}$$

Where $E$and$G$ are the elastic constants of the plate material E and$G$ are young's modulus and shear modulus or the elastic constants of the plate material, and$\alpha$is the coefficients of thermal expansion in x and z direction respectively and$T_0$and$T_1$is the thermal load.

1.2.6 Governing equations and boundary conditions

Governing differential equations and boundary conditions are obtained from Principle of virtual work. Using equations for stresses, strains and principle of virtual work, variationallyconsistent differential equations for plate under consideration are obtained. The principle of virtual work when applied to plate leads to:

$$\int_{z=-h/2}^{z=h/2}\int_{y=0}^{y=b}\int_{x=0}^{x=a}\left(\sigma_x\delta\varepsilon_x+\sigma_y\delta\varepsilon_y+\tau_{yz}\delta\varepsilon_{yz}+\tau_{zx}\delta\varepsilon_{zx}+\tau_{xy}\delta\varepsilon_{xy}\right)dx\,dy - \int_{y=0}^{y=b}\int_{x=0}^{x=a} q(x,y)\,\delta w\,dx\,dy \tag{10}$$

Whereδ = variational operator,
Substituting expressions for the strains and stresses in Eq. (3.10)and employingGreen's theorem successively, we obtain the coupled Euler-Lagrange equations which are the governing differential equations and associated boundary conditions of the plate. The governing differential equations obtained are as follows:

$$\delta w:\ D_{11}\frac{\partial^4 w}{\partial x^4} + 2\left(D_{12}+2D_{66}\right)\frac{\partial^4 w}{\partial x^2\partial x^2} + D_{22}\frac{\partial^4 w}{\partial y^4} - \left(S_{11}\frac{\partial^3\phi}{\partial x^3}+S_{22}\frac{\partial^3\psi}{\partial x^3}\right)-\left(S_{12}+2S_{66}\right)\left(\frac{\partial^3\phi}{\partial x\partial y^2}+\frac{\partial^3\psi}{\partial x^2\partial y}\right)+ \left(TD_{11}+TTD_{12}\right)\frac{\partial^2 T_1}{\partial x^2} + \left(TD_{12}+TTD_{22}\right)\frac{\partial^2 T_1}{\partial y^2} = q, \tag{11}$$

$$\delta\phi:\ S_{11}\frac{\partial^3 w}{\partial x^3} + \left(S_{12}+2S_{66}\right)\frac{\partial^3 w}{\partial x\partial y^2} - \left(SS_{11}\frac{\partial^2\phi}{\partial x^2}+SS_{66}\frac{\partial^2\phi}{\partial y^2}\right) + C_{55}\phi - \left(SS_{12}+SS_{66}\right)\frac{\partial^2\psi}{\partial y\partial x} + \left(TS_{11}+TTS_{12}\right)\frac{\partial T_1}{\partial x} = 0, \tag{12}$$

$$\delta\psi:\ S_{22}\frac{\partial^3 w}{\partial y^3} + \left(S_{12}+2S_{66}\right)\frac{\partial^3 w}{\partial y\partial x^2} - \left(SS_{66}\frac{\partial^2\psi}{\partial x^2}+SS_{22}\frac{\partial^2\psi}{\partial y^2}\right) + C_{44}\psi - \left(SS_{12}+SS_{66}\right)\frac{\partial^2\phi}{\partial y\partial x} + \left(TS_{12}+TTS_{22}\right)\frac{\partial T_1}{\partial y} = 0, \tag{13}$$

The associate consistent boundary conditions obtained are as below: Along the edge $x = 0$ and $x = a$,

$$-D_{22}\frac{\partial^3 w}{\partial y^3}-\left(D_{12}+4D_{66}\right)\frac{\partial^3 w}{\partial y\partial x^2}+\left(2S_{66}\frac{\partial^2\psi}{\partial x^2}+S_{66}\frac{\partial^2\psi}{\partial y^2}\right) + \left(S_{12}+2S_{66}\right)\frac{\partial^2\phi}{\partial y\partial x} - \left(TD_{12}+TTD_{22}\right)\frac{\partial T_1}{\partial y} = 0 \quad \text{or } w \text{ is prescribed} \tag{14}$$

$$\left(D_{12}\frac{\partial^2 w}{\partial x^2}+D_{22}\frac{\partial^2 w}{\partial y^2}\right)-S_{12}\frac{\partial\phi}{\partial x} - S_{22}\frac{\partial\psi}{\partial y}+\left(TD_{12}+TTD_{22}\right)T_1 = 0 \quad \text{or } \frac{\partial w}{\partial x} \text{ is prescribed} \tag{15}$$

$$SS_{66}\left(\frac{\partial\phi}{\partial y}+\frac{\partial\psi}{\partial x}\right)-2S_{66}\frac{\partial^2 w}{\partial y\partial x}=0 \quad \text{or } \phi \text{ is prescribed} \tag{16}$$

$$S_{12}\frac{\partial^2 w}{\partial x^2}+SS_{12}\frac{\partial\phi}{\partial x}-S_{22}\frac{\partial^2 w}{\partial y^2}+ SS_{22}\frac{\partial\psi}{\partial y}\left(TS_{12}+TTS_{22}\right)T_1 = 0 \quad \text{or } \psi \text{ is prescribed} \tag{17}$$

Along the edge $y = 0$ and $x = b$,

$$-D_{11}\frac{\partial^3 w}{\partial x^3}-\left(D_{12}+4D_{66}\right)\frac{\partial^3 w}{\partial x\partial y^2}+\left(2S_{66}\frac{\partial^2\phi}{\partial y^2}+S_{66}\frac{\partial^2\phi}{\partial x^2}\right) + \left(S_{12}+2S_{66}\right)\frac{\partial^2\psi}{\partial y\partial x} - \left(TD_{12}+TTD_{22}\right)\frac{\partial T_1}{\partial x} = 0 \quad \text{or } w \text{ is prescribed} \tag{18}$$

$$\left(D_{12}\frac{\partial^2 w}{\partial x^2}+D_{22}\frac{\partial^2 w}{\partial y^2}\right) - S_{11}\frac{\partial\phi}{\partial x} - S_{12}\frac{\partial\psi}{\partial y} + \left(TD_{12}+TTD_{22}\right)T_1 = 0 \quad \text{or } \frac{\partial w}{\partial x} \text{ is prescribed} \tag{19}$$

$$-\left(S_{11}\frac{\partial^2 w}{\partial x^2}+S_{12}\frac{\partial^2 w}{\partial y^2}\right) + SS_{11}\frac{\partial\phi}{\partial x} + SS_{12}\frac{\partial\psi}{\partial y} -\left(TS_{11}+TTS_{12}\right)T_1=0 \quad \text{or } \phi \text{ is prescribed} \tag{20}$$

$$SS_{66}\left(\frac{\partial\psi}{\partial x}+\frac{\partial\phi}{\partial y}\right) - 2S_{66}\frac{\partial^2 w}{\partial x\partial y} = 0 \quad \text{or } \psi \text{ is prescribed} \tag{21}$$

Thus, the variationally consistent governing differential equations and boundary conditionsare obtained. The coefficients appearing in the governing differential equations and boundaryconditions are given in Appendix. The flexural behaviour of the plate is described by thesolutionsatisfying these equations and the associated boundary conditions at each edge of theplate.

1.2.7 The general solution of governing equilibrium equations of the plate:

To assess the performance of the present theory in the prediction of bending response of a plate under a thermal load, a simply supported isotropic plate of length *L*, width *b*, and thickness *h* is considered. The plate subjected to a thermal load $T(x, y) = zT_1(x, y)$ varies linearly through the thickness.

$$\{T_1\} = \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \left\{ T_{1mn} \sin\frac{m\pi x}{a} \sin\frac{m\pi y}{b} \right\} \qquad (22)$$

Where, T_{1mn} is the coefficient of Fourier expansion

$$T_{1mn} = \frac{16T_0}{mn\pi^2} \quad \text{for} \quad m = 1, 3, 5, ... \text{ and } \quad n = 2, 4, 6, ...,$$

$$T_{1mn} = 0 \quad \text{for} \quad n = 1, 3, 5, ... \text{ and } \quad m = 2, 4, 6, ...,\qquad (23)$$

The quantities T_0 are the intensities of thermal load. In order to solve the governing equation with the prescribed boundary conditions; a generalized Navier approach is employed to obtain closed-form solutions. The following is the solution form for *w(x,y)*, *ϕ(x, y)*, and *ψ(x, y)* satisfying the boundary conditions of simply supported plate as given below

$$w = \psi = M_x - M_x^s = 0 \text{ at } x = 0 \text{ and } x = a,$$

$$w = \phi = M_y - M_y^s = 0 \text{ at } y = 0 \text{ and } y = b.$$

Consider the mechanical load is zero. *i.e.* $q_o = 0$

$$\begin{Bmatrix} w \\ \phi \\ \psi \end{Bmatrix} = \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \begin{Bmatrix} w_{mn} \sin\frac{m\pi x}{a} \sin\frac{m\pi y}{b} \\ \phi_{mn} \cos\frac{m\pi x}{a} \sin\frac{m\pi y}{b} \\ \psi_{mn} \sin\frac{m\pi x}{a} \cos\frac{m\pi y}{b} \end{Bmatrix} \qquad (24)$$

Wherew_{mn}, ϕ_{mn}, and$\psi_{mn}$are the unknown coefficients which can be easily determined by substituting Eq. (16) and$T(x, y) = zT_1(x, y)$ in the set of three governing differential equation and solving the resulting simultaneous equation as,

$$K_{11} w_{mn} + K_{12} \phi_{mn} + K_{13} \psi_{mn} = F_1$$

$$K_{21} w_{mn} + K_{22} \phi_{mn} + K_{23} \psi_{mn} = F_2$$

$$K_{31} w_{mn} + K_{32} \phi_{mn} + K_{33} \psi_{mn} = F_3 \qquad (25)$$

Where,

$$K_{11} = D_{11} \frac{m^4\pi^4}{a^4} + 2D_{12} \frac{m^2n^2\pi^2}{a^2b^2} + D_{22} \frac{n^4\pi^4}{b^4} + 4D_{66} \frac{m^2n^2\pi^2}{a^2b^2},$$

$$K_{12} = -S_{11} \frac{m^3\pi^3}{a^3} - \left(S_{12} + 2S_{66}\right) \frac{mn^2\pi^3}{ab^2},$$

$$K_{13} = -S_{22} \frac{n^3\pi^3}{b^3} - \left(S_{12} + 2S_{66}\right) \frac{m^2n\pi^3}{a^2b},$$

$$K_{22} = SS_{11} \frac{m^2\pi^2}{a^2} + SS_{66} \frac{n^2\pi^2}{b^2} + C_{55},$$

$$K_{23} = \left(SS_{12} + SS_{66}\right) \frac{mn\pi^2}{ab},$$

$$K_{33} = SS_{22} \frac{n^2\pi^2}{b^2} + SS_{66} \frac{m^2\pi^2}{a^2} + C_{44},$$

$$F_1 = \left(D_{12}\alpha_x + D_{22}\alpha_y\right) \frac{n^2\pi^2}{b^2} T_{1mn} + \left(D_{11}\alpha_x + D_{12}\alpha_y\right) \frac{m^2\pi^2}{a^2} T_{1mn},$$

$$F_2 = -\left(S_{11}\alpha_x + S_{12}\alpha_y\right) \frac{m\pi}{a} T_{1mn},$$

$$F_3 = -\left(S_{12}\alpha_x + S_{22}\alpha_y\right) \frac{n\pi}{b} T_{1mn}.$$

Thermal Analysis:

A simply supported plate of length*a*, width *b*, and thickness *h* of a homogeneous isotropic material is considered. The plate occupies the region described by Eqn. (1) in *x-y-z* Cartesian coordinate system.

1.2.8 Illustrative examples

In order to prove the efficiency of the present theory, the following numerical example is considered. The following material properties for plate are used.

Material properties:

1. Modulus of Elasticity *E* = 210 GPa

2. Poisson'sRatio $\mu = 0.30$

3. Coefficient of Thermal Expansion

$\alpha_x = \alpha_x = 12\text{X}10^{-6}/°\text{C}$

The kinematic and static (forced) boundary conditions associated with various plate bending problems depending upon type of supports are as follows:Simply Supported Isotropic Square Plate ($a=b$)

1.2.9 Mathematical formulation

Expression for axial displacement (w)

$$w=w_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \qquad (26)$$

Expression for axial displacement (ϕ)

$$\phi=\phi_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \qquad (27)$$

Expression for axial displacement (ψ)

$$\psi=\psi_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) \qquad (28)$$

Substituting expressions for w, ϕ and ψ given by Eqn. (26), (27) and (28) into Eqns. (2), (3), and (8), the final expressions for axial displacement $\overline{u}$,axial displacement $\overline{v}$, axial stresses $\overline{\sigma}_x, \overline{\sigma}_y$ andtransverse shear stress $\overline{\tau}_{xy}, \overline{\tau}_{yz}, \overline{\tau}_{xz}$ can be obtained respectively.

Expression for axial displacement (u)

$$u=\left\{-z\, w_{mn}\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)+\left[z-\left(\frac{h}{\pi}\right)\sin\left(\frac{\pi z}{h}\right)\right]\phi_{xy}\right\} \qquad (29)$$

Expression for axial displacement (v)

$$v=\left\{-z\, w_{mn}\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)+\left[z-\left(\frac{h}{\pi}\right)\sin\left(\frac{\pi z}{h}\right)\right]\psi_{xy}\right\} \qquad (30)$$

Expression for axial stress (σ_x)

$$\sigma_x=\left\{\begin{array}{l} zw_{mn}\left[\frac{m^2\pi^2}{a^2}+\mu\frac{n^2\pi^2}{b^2}\right]-\left[z-\left(\frac{h}{\pi}\right)\sin\left(\frac{\pi z}{h}\right)\right]\left[\phi_{mn}\frac{m\pi}{a}+\mu\psi_{mn}\frac{n\pi}{b}\right] \\ -zT_1mn\left(\alpha_x+\mu\alpha_y\right)\end{array}\right\}$$

$$\text{x}\ \frac{E}{\left(1-\mu^2\right)}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \qquad (31)$$

Expression for axial stress (σ_y)

$$\sigma_y=\left\{\begin{array}{l} zw_{mn}\left[\mu\frac{m^2\pi^2}{a^2}+\frac{n^2\pi^2}{b^2}\right]-\left[z-\left(\frac{h}{\pi}\right)\sin\left(\frac{\pi z}{h}\right)\right]\left[\mu\phi_{mn}\frac{m\pi}{a}+\psi_{mn}\frac{n\pi}{b}\right] \\ -zT_1mn\left(\mu\alpha_x+\alpha_y\right)\end{array}\right\}$$

$$\frac{E}{\left(1-\mu^2\right)}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \qquad (32)$$

Expression for transverse shear stress ($\boldsymbol{\tau}_{xy}$)

$$\tau_{xy}=\left\{-2zw_{mn}\frac{mn\pi^2}{ab}+\left[z-\left(\frac{h}{\pi}\right)\sin\left(\frac{\pi z}{h}\right)\right]\left[\phi_{mn}\frac{n\pi}{b}+\psi_{mn}\frac{m\pi}{a}\right]\right\}$$

$$\frac{E}{2(1+\mu)}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) \qquad (33)$$

The alternate approach to determine the transverse shear stresses is the use of equilibrium equations. Integrating the below two equilibrium equation with respect to the thickness coordinate and satisfying the boundary conditions at the bounding surfaces of the plate on can obtain the final expressions of transverse shear stresses. The stress equilibrium equations are as follows:

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}=0 \qquad (34a)$$

$$\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau_{zy}}{\partial z}=0 \qquad (34b)$$

Expression for transverse shear stress ($\boldsymbol{\tau}_{yz}$)

$$\tau_{yz}=\left\{\begin{array}{l} \frac{E}{\left(1-\mu^2\right)}\frac{n\pi}{b}\left[\begin{array}{l} w_{mn}\left(\frac{h^2}{8}-\frac{z^2}{2}\right)\left(\mu\frac{m^2\pi^2}{a^2}+\frac{n^2\pi^2}{b^2}\right)+ \\ \left(\frac{z^2}{2}-\frac{h^2}{8}+\frac{h^2}{\pi^2}\left[\cos\left(\frac{\pi z}{h}\right)-\cos\left(\frac{\pi}{2}\right)\right]\right)\end{array}\right] \\ \left(\mu\phi_{mn}\frac{m\pi}{a}+\psi_{mn}\frac{n\pi}{b}\right)+T_1mn\left(\mu\alpha_x+\alpha_y\right)\left(\frac{z^2}{2}-\frac{h^2}{8}\right) \\ +\frac{E}{(1+\mu)}\frac{m\pi}{a}\left[w_{mn}\frac{mn\pi^2}{ab}\left(\frac{h^2}{8}-\frac{z^2}{2}\right)\right]+\frac{1}{2}\left(\begin{array}{l}\frac{z^2}{2}-\frac{h^2}{8}+\frac{h^2}{\pi^2} \\ \left[\cos\left(\frac{\pi z}{h}\right)-\cos\left(\frac{\pi}{2}\right)\right]\end{array}\right) \\ \left(\phi_{mn}\frac{n\pi}{b}+\psi_{mn}\frac{m\pi}{a}\right)\end{array}\right\}$$

$$\text{x}\ \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) \qquad (35)$$

Expression for transverse shear stress ($\boldsymbol{\tau}_{xz}$)

$$\tau_{zx}=\left\{\begin{array}{l} \frac{E}{\left(1-\mu^2\right)}\frac{m\pi}{a}\left[\begin{array}{l} w_{mn}\left(\frac{h^2}{8}-\frac{z^2}{2}\right)\left(\frac{m^2\pi^2}{a^2}+\mu\frac{n^2\pi^2}{b^2}\right)+ \\ \left(\frac{z^2}{2}-\frac{h^2}{8}+\frac{h^2}{\pi^2}\left[\cos\left(\frac{\pi z}{h}\right)-\cos\left(\frac{\pi}{2}\right)\right]\right)\end{array}\right] \\ \left(\phi_{mn}\frac{m\pi}{a}+\mu\psi_{mn}\frac{n\pi}{b}\right)+T_1mn\left(\alpha_x+\mu\alpha_y\right)\left(\frac{z^2}{2}-\frac{h^2}{8}\right) \\ +\frac{E}{(1+\mu)}\frac{n\pi}{b}\left[w_{mn}\frac{mn\pi^2}{ab}\left(\frac{h^2}{8}-\frac{z^2}{2}\right)\right]+ \\ \frac{1}{2}\left(\phi_{mn}\frac{n\pi}{b}+\psi_{mn}\frac{m\pi}{a}\right)\left(\frac{z^2}{2}-\frac{h^2}{8}\right)+\frac{h^2}{\pi^2}\left[\cos\left(\frac{\pi z}{h}\right)-\cos\left(\frac{\pi}{2}\right)\right]\end{array}\right\}$$

$$\text{x}\ \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \qquad (36)$$

2. RESULTS AND DISCUSSION

2.1 Numerical Calculation

In this chapter, the results for maximum transverse displacement $\overline{w}$, in-plane displacements $\overline{u}$ and $\overline{v}$, in-plane normal stress components $\overline{\sigma}_x$and $\overline{\sigma}_y$,the in-plane shear stress components $\overline{\tau}_{yx}$ transverse shear stress components $\overline{\tau}_{yz}$and$\overline{\tau}_{zx}$are presented in the following non dimensional form for the purpose of presenting the results in this work.

For plate subjected to sinusoidal thermal load.

$$\bar{u} = \frac{1}{\left(\alpha_1 T_0 E_2 b^2\right)} u\left(0, \frac{b}{2}, \pm\frac{h}{2}\right),$$

$$\bar{v} = \frac{1}{\left(\alpha_1 T_0 E_2 b^2\right)} v\left(\frac{a}{2}, 0, \pm\frac{h}{2}\right),$$

$$\bar{w} = \frac{10}{\left(\alpha_1 T_0 E_2 b^2\right)} w\left(\frac{a}{2}, \frac{b}{2}, \pm\frac{h}{2}\right),$$

$$\bar{\sigma}_x, \bar{\sigma}_y = \frac{1}{\left(\alpha_1 T_0 E_2 b^2\right)} \left(\sigma_x, \sigma_y\right)\left(\frac{a}{2}, \frac{b}{2}, \pm\frac{h}{2}\right),$$

$$\bar{\tau}_{xy} = \frac{1}{\left(\alpha_1 T_0 E_2 b^2\right)} \left(\tau_{xy}\right)\left(0, 0, \pm\frac{h}{2}\right),$$

$$\bar{\tau}_{xz} = \frac{10}{\left(\alpha_1 T_0 E_2 b^2\right)} \left(\tau_{xz}\right)\left(0, \frac{b}{2}, 0\right),$$

$$\bar{\tau}_{yz} = \frac{10}{\left(\alpha_1 T_0 E_2 b^2\right)} \left(\tau_{yz}\right)\left(\frac{a}{2}, 0, 0\right).$$

All the parameters are obtained by solving the force matrix and equilibrium equations.

2.2 Numerical Result for Simply Supported Square plate (*a*=*b*) is the simply supported thick square plate, which is made up of isotropic material. The properties of plate are given below.

1) Coefficient of Thermal Expansion

$\alpha_x = \alpha_y = 12\text{x}10^{-6}$/°C

2) Youngs'sModulus *E*= 210GPa

3) Poissons Ratio μ=0.3, Hence the plate is square, *a*=*b*

Table 4.1:Maximum transverse displacement $\overline{w}$ at *(x =a/2* and *y = b/2*), in-plane displacement components $\overline{u}$and $\overline{v}$ at *(x = 0, y = b/2* and *z = h/2)* and *(x = a/2, y = 0* and *z = h/2)* respectively, of simply supported rectangular plate (*a* = *b*) subjected to sinusoidal thermal load for Aspect Ratio 5.

Source	Model	$\overline{w}$	$\overline{u}$	$\overline{v}$
Present	TSDT	0.5338	0.0335	0.0335
Shinde	HSDT	0.5338	0.0335	0.0335
Kirchoff - Love	CPT	0.6470	0.0406	0.0406

Table 4.2: Normal stress components $\overline{\sigma}_x$and $\overline{\sigma}_y$at *(x = a/2, y = b/2* and *z = h/2)*, of simply supported rectangular plate (*a* = *b*) subjected to sinusoidal thermal load for Aspect Ratio 5.

Source	Model	$\overline{\sigma}_x$	$\overline{\sigma}_y$
Present	TSDT	0.0810	0.0810
Shinde	HSDT	0.0810	0.0810
Kirchoff - Love	CPT	0.0491	0.0491

Table 4.3: in-plane shear stress components $\overline{\tau}_{xy}$at *(x = 0, y = 0* and *z = h/2)*, transverse shear stress components $\overline{\tau}_{yz}$at(*x = a/2, y = 0* and *z = 0)*,transverse shear stress components $\overline{\tau}_{zx}$at *(x = 0, y = b/2* and *z = 0)* of simply supported rectangular plate (*a* = *b*) subjected to sinusoidal thermal load for Aspect Ratio 5.

Source	Model	$\overline{\tau}_{xy}$	$\overline{\tau}_{yz}$	$\overline{\tau}_{zx}$
Present	TSDT	0.0810	-0.0254	-0.0254
Shinde	HSDT	0.0810	-0.0254	-0.0254
Kirchoff - Love	CPT	0.0981	---	---

Table 4.4: Maximum transverse displacement $\overline{w}$ at *(x =a/2* and *y = b/2*), in-plane displacement components $\overline{u}$and $\overline{v}$ at *(x = 0, y = b/2* and *z = h/2)* and *(x = a/2, y = 0* and *z = h/2)* respectively, of simply supported rectangular plate (*a* = *b*) subjected to sinusoidal thermal load for Aspect Ratio 10.

Source	Model	$\overline{w}$	$\overline{u}$	$\overline{v}$
Present	TSDT	1.0676	0.0167	0.0167
Shinde	HSDT	1.0676	0.0167	0.0167
Kirchoff - Love	CPT	1.2941	0.0203	0.0203

Table 4.5: Normal stress components $\overline{\sigma}_x$and $\overline{\sigma}_y$at *(x = a/2, y = b/2* and *z = h/2)*, of simply supported rectangular plate (*a* = *b*) subjected to sinusoidal thermal load for Aspect Ratio 10.

Source	Model	$\overline{\sigma}_x$	$\overline{\sigma}_y$
Present	TSDT	0.0405	0.0405

Shinde	HSDT	0.0405	0.0405
Kirchoff - Love	CPT	0.0245	0.0245

Table 4.6: in-plane shear stress components $\overline{\tau}_{xy}$at *(x = 0, y = 0* and *z = h/2)*, transverse shear stress components $\overline{\tau}_{yz}$at*(x = a/2, y = 0* and *z = 0)*, transverse shear stress components $\overline{\tau}_{zx}$at *(x = 0, y = b/2* and *z = 0)* of simply supported rectangular plate (*a* = *b*) subjected to sinusoidal thermal load for Aspect Ratio 10.

Source	Model	$\overline{\tau}_{xy}$	$\overline{\tau}_{yz}$	$\overline{\tau}_{zx}$
Present	TSDT	0.0405	-0.0063	-0.0063
Shinde	HSDT	0.0405	-0.0063	-0.0063
Kirchoff - Love	CPT	0.0491	---	---

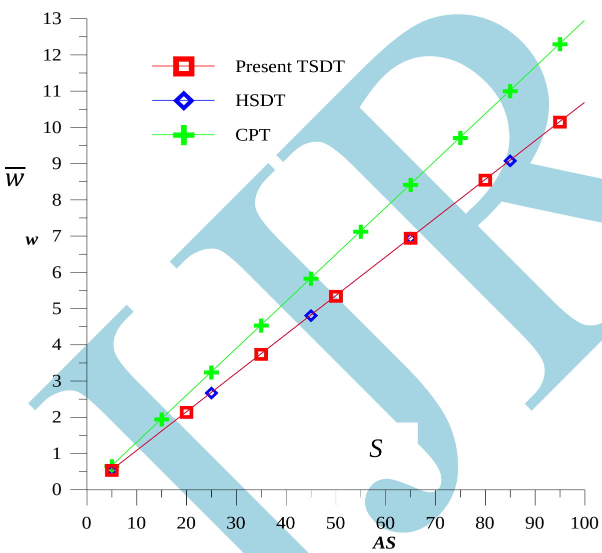

Figure 2.1: Variation of maximum transverse displacement $\overline{w}$ in *z* direction of simply supported square plate at *(x =a/2* and *y = b/2)* when subjected to thermal load with aspect ratio (*S*).

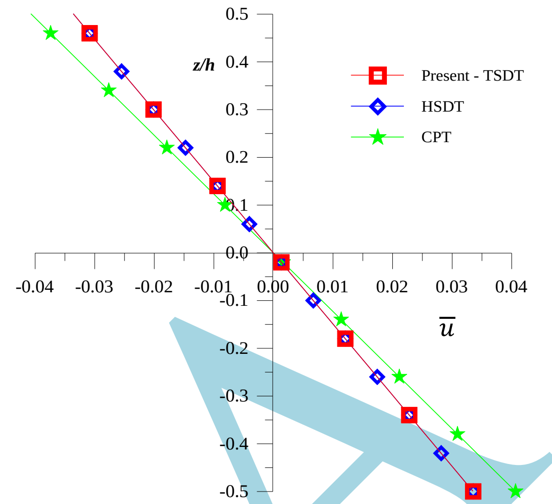

Figure 2.2: Variation, throughout the thickness, of the in-plane displacement components $\overline{u}$ in the *x* directions of simply supported square plate at *(x = 0, y = b/2* and *z = h/2)* when subjected to thermal load for aspect ratio 5.

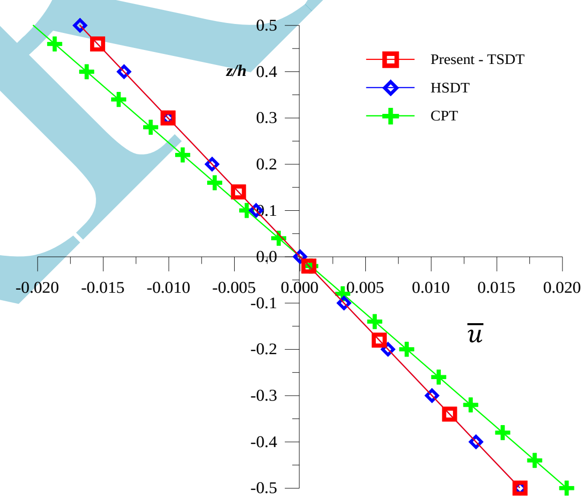

Figure 2.3: Variation, throughout the thickness, of the in-plane displacement components $\overline{u}$ in the *x* directions of simply supported square plate at *(x = 0, y = b/2* and *z = h/2)* when subjected to thermal load for aspect ratio 10.

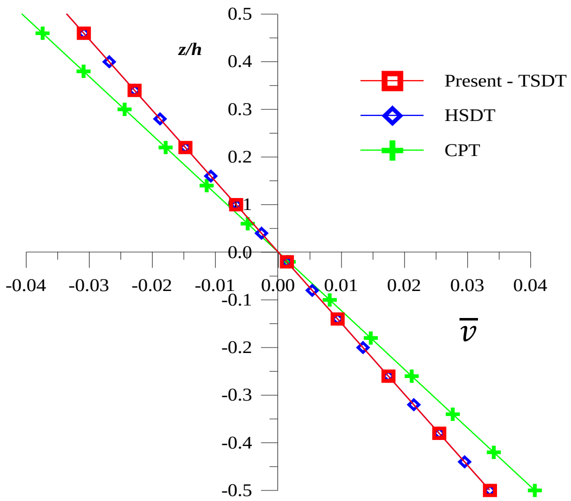

Figure 2.4: Variation, throughout the thickness, of the in-plane displacement components $\bar{v}$ in the y directions of simply supported square plate at *(x = a/2, y = 0* and *z = h/2)* when subjected to thermal load for aspect ratio 5. (Numerical 1)

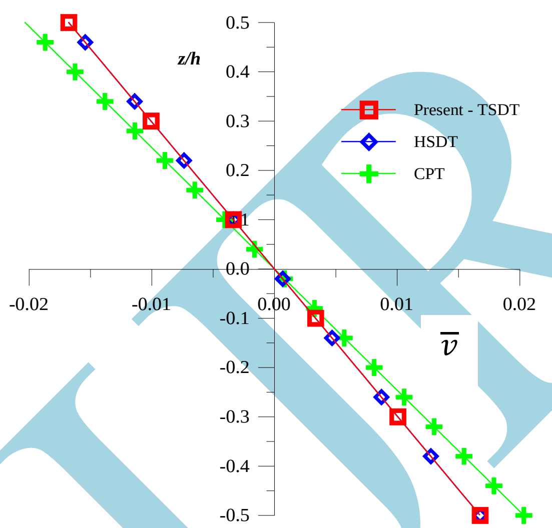

Figure 2.5: Variation, throughout the thickness, of the in-plane displacement components $\bar{v}$ in the y directions of simply supported square plate at *(x = a/2, y = 0* and *z = h/2)* when subjected to thermal load for aspect ratio 10. (Numerical 1)

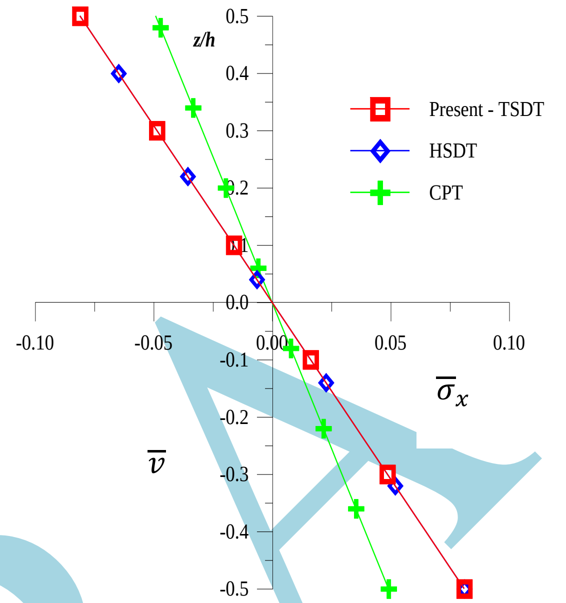

Figure 2.6: Variation, throughout the thickness, of the in-plane normal stress components $\bar{\sigma}_x$ in the x directions of simply supported square plate at *(x = a/2, y = b/2* and *z = h/2)* when subjected to thermal load for aspect ratio 5.

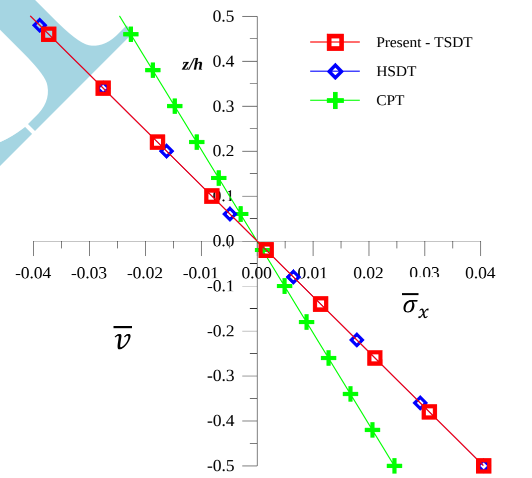

Figure 2.7: Variation, throughout the thickness, of the in-plane normal stress components $\bar{\sigma}_x$ in the x directions of simply supported square plate at *(x = a/2, y = b/2* and *z = h/2)* when subjected to thermal load for aspect ratio 10.

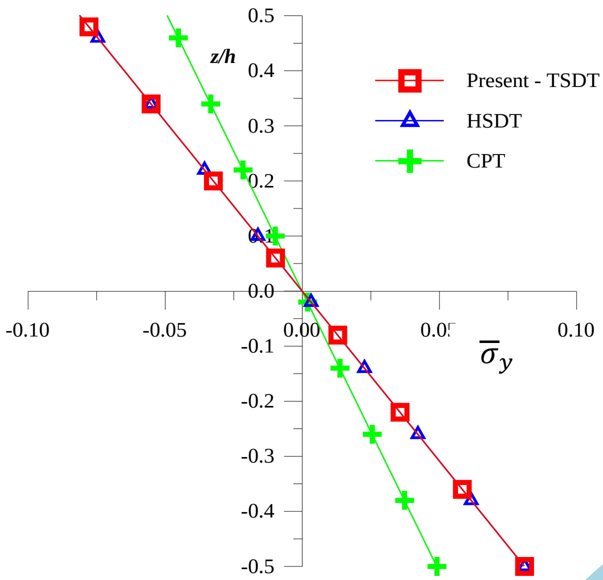

Figure 2.8: Variation, throughout the thickness, of the in-plane normal stress components $\overline{\sigma}_y$ in the y directions of simply supported square plate at *(x = a/2, y = b/2* and *z = h/2)* when subjected to thermal load for aspect ratio 5. (Numerical 1)

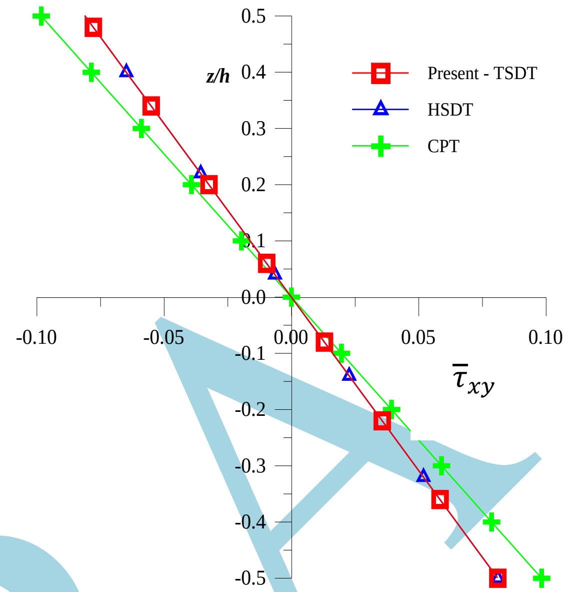

Figure 2.10: Variation, throughout the thickness, of the in-plane shear stress components $\overline{\tau}_{xy}$ of simply supported square plate at *(x = 0, y = 0* and *z = h/2)* when subjected to thermal load for aspect ratio 5.

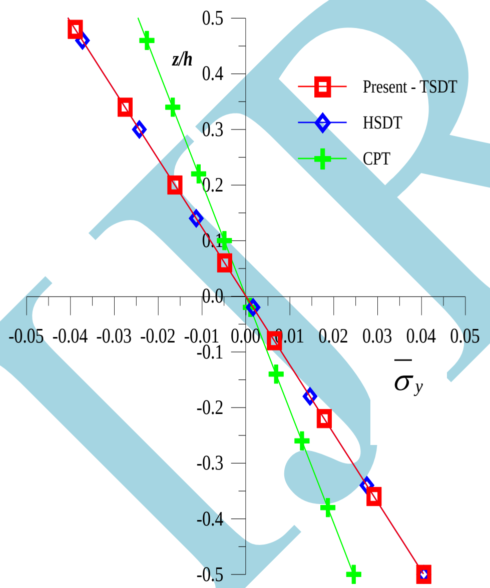

Figure 2.9: Variation, throughout the thickness, of the in-plane normal stress components $\overline{\sigma}_y$ in the y directions of simply supported square plate at *(x = a/2, y = b/2* and *z = h/2)* when subjected to thermal load for aspect ratio 10. (Numerical 1)

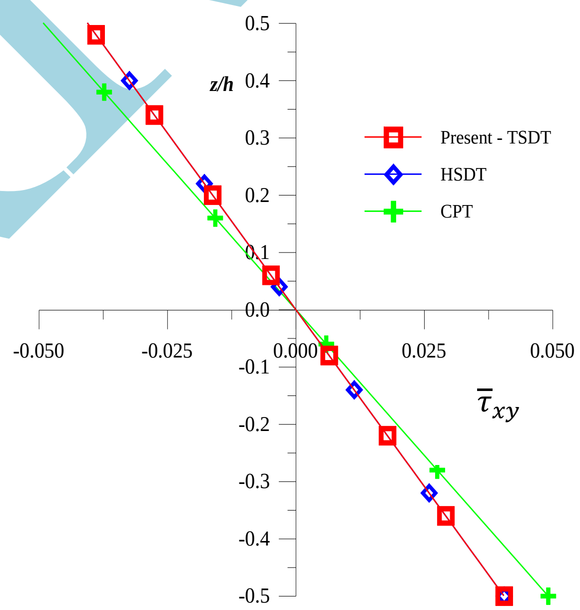

Figure 2.11: Variation, throughout the thickness, of the in-plane shear stress components $\overline{\tau}_{xy}$ of simply supported square plate at *(x = 0, y = 0* and *z = h/2)* when subjected to thermal load for aspect ratio 10.

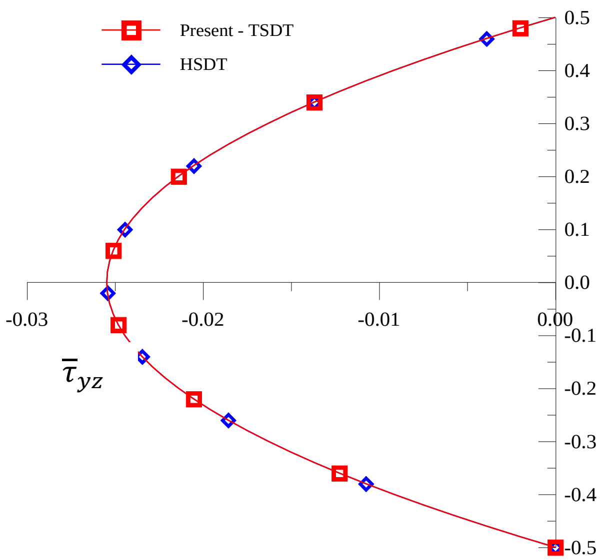

Figure 2.12: Variation, throughout the thickness, of the transverse shear stress components $\bar{\tau}_{yz}$ of simply supported square plate at *(x = a/2, y = 0* and *z = 0)* when subjected to thermal load for aspect ratio 5.

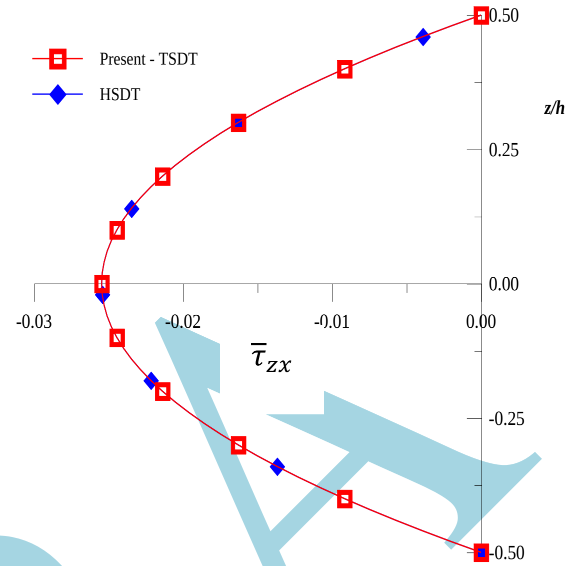

Figure 2.14: Variation of the transverse shear stress components $\bar{\tau}_{zx}$ of simply supported square plate at *(x = 0, y = b/2* and *z = 0)* when subjected to thermal load for aspect ratio 5.

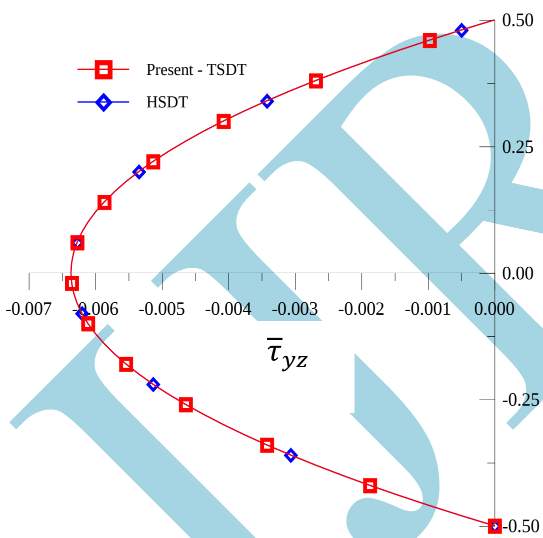

Figure 2.13: Variation, throughout the thickness, of the transverse shear stress components $\bar{\tau}_{yz}$ of simply supported square plate at *(x = a/2, y = 0* and *z = 0)* when subjected to thermal load for aspect ratio 10.

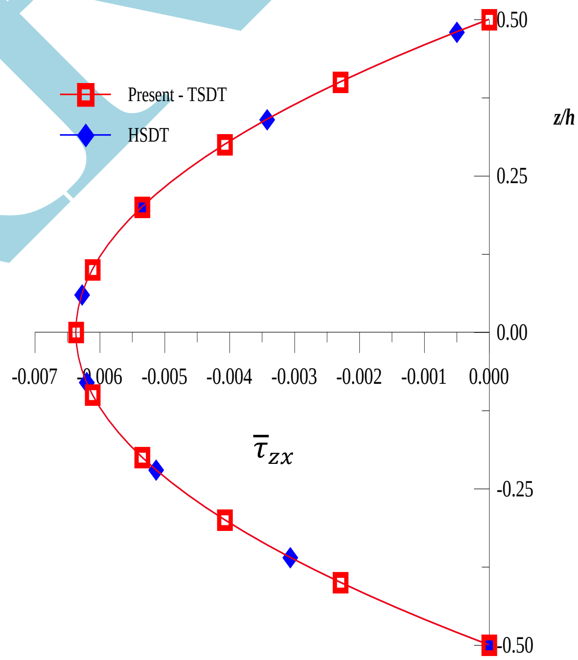

Figure 2.15: Variation of the transverse shear stress components $\bar{\tau}_{zx}$ of simply supported square plate at *(x = 0, y = b/2* and *z = 0)* when subjected to thermal load for aspect ratio 10.

2.5. Discussion of Results

In order to validate the efficiency of the present Trigonometric shear deformation, a sample problems is considered. The results obtained for displacements and stresses are compared with the corresponding results of the classical plate theory (CPT), higher order shear deformation theory (HSDT). The comparison of results of maximum non-dimensional axial and transverse displacements, axial and shear stresses for aspect ratios of 5 and 10 are presented for plate subjected to thermal loads. The results in this section are discussed below.

2.5.1 Transverse displacement $\overline{w}$

Numerical results are obtained for various *b*/*h* and *a*/*b* ratios. It can be observed that the results obtained by the present theory are in excellent agreement with those of HSDT, whereas CPT underestimates the transverse displacements for all aspect ratios *b*/*h* with respect to that of the present theory. The results obtained by using the present theory agree extremely well with those obtained by HSDT. CPT under-predicts the transverse displacement $\overline{w}$. The variation of transverse displacement $\overline{w}$ for aspect ratio 5 and 10 is presented in Table (2.1), (2.2), (2.3), (2.4), (2.5) and (2.6) and Fig. (2.1) for simply supported plate with sinusoidal thermal load. The displacement predicted by CPT is lower than that of TSDT and HSDT. Transverse displacement obtained for isotropic plate by present theory is in good agreement with higher order shear deformation theory.

2.5.2 The in-plane displacement $\overline{u}$and$\overline{v}$

The in-plane displacement $\overline{u}$ and $\overline{v}$ in the *x* and *y* directions respectively for aspect ratio 5 and 10 is presented in Table (2.1), (2.2), (2.3), (2.4), (2.5) and (2.6) and Fig. (2.6) (2.7), (2.8), (2.9).The displacement predicted by CPT is lower than that of TSDT and HSDT.

1. The values of axial displacement predicted by HSDT and TSDT are identical for all aspect ratios. The through thickness distribution of this displacement obtained by present theory is in close agreement with other refined theories except the CPT.
2. There is considerable variation in result for *a/b* ratio, for *a/b = 1* the values for transverse shear stresses are same for all aspect ratio.

4.5.3 The in-plane normal stress components $\overline{\sigma}_x$and$\overline{\sigma}_y$

The in-plane normal stress components $\overline{\sigma}_x$and$\overline{\sigma}_y$ in the *x* and *y* directions respectively for aspect ratio 5 and 10 is presented in Table (2.1), (2.2), (2.3), (2.4), (2.5) and (2.6) and Fig. (2.6) (2.7), (2.8), (2.9).The result predicted by CPT is lower than that of TSDT and HSDT.

1. It is observed that the results by present theory are in excellent agreement with other refined theories. However, CPT yield lower values of this stress as compared to the values given by other refined theories. The through the thickness variation of this stress given by CPT is linear throughout the thickness of plate, indicating the effect of neglect of shear deformation.
2. Present and FSDT provide the non-linear variations of axial stress across the thickness at the built-in end due to heavy stress concentration. However, this effect of local stress concentration cannot be captured by lower order theory like CPT.
3. There is considerable variation in result for *a/b* ratio, for *a/b = 1* the values for transverse shear stresses are same for all aspect ratio.

2.5.4 The in-plane shear stress $\overline{\tau}_{xy}$

The In-plane shear stresses $\overline{\tau}_{xy}$ obtained for isotropic plate by present theory are comparable with HSDT and CPT, whereas CPT underestimates it compared to the results of present theory and HSDT. The in-plane shear stresses $\overline{\tau}_{xy}$ for aspect ratio 5 and 10 is presented in Table (2.1), (2.2), (2.3), (2.4), (2.5) and (2.6) and Figs. (2.10), (2.11), The result obtained by CPT is quite lower than that of TSDT and HSDT.

4.5.5 The transverse shear stress$\overline{\tau}_{yz}$and $\overline{\tau}_{zx}$: The Transverse Shear Stress$\overline{\tau}_{yz}$and$\overline{\tau}_{zx}$ for aspect ratio 5 and 10 is presented in Table (2.1), (2.2), (2.3), (2.4), (2.5) and (2.6) and Figs. (2.12), (2.13), (2.14) and (2.15). The Transverse Shear Stress is obtained by equilibrium equation.Here, the realistic variation means the variation given by any refined theory which resembles to that of elasticity solution. The transverse shear stress obtained by both the theories satisfies the shear stress free conditions on the top and bottom surfaces of the plate.

1. For simply supported plate maximum transverse shear stress obtained by present theory is in close agreement with that of other higher order theory (HSDT). The values of present theory and those of HSDT are in good agreement with each other.
2. The maximum negative value of this stress occurs at the neutral axis.

There is considerable variation in result for *a/b* ratio, for *a/b = 1* .

REFERENCES

1. Reddy J. N., 2004, Mechanics of laminated composite plates and shell theory and analysis.
2. Boley B. A. and Weiner J. H., John Wiley, 1960, "Theory of Thermal Stresses," New York.
3. B. M. Shinde, A. S. Sayyad and A. B. Kawade, 2013, "Thermal analysis of isotropic plates using hyperbolic shear deformation theory," Applied and Computational Mechanics, Vol. 7, pp. 193–204
4. Cinelli G., 1965, "An Extension of the Finite Hankel Transform and Applications," Int. J. Engng Sci. Vol. 3, pp. 539-559.
5. Shinde B. M., Kawade A. B. and Sayyad A. S., 2013, "Thermal Response of Isotropic Plates Using Hyperbolic Shear Deformation Theory" International Journal of Advanced Technology in Civil Engineering, Vol. 2, Issue-1, pp. 140-145.
6. Atteshamuddin S. Sayyad and Ghugal Y. M., 2015, "On the free vibration analysis of laminated composite and sandwich plates: A review of recent literature with some numerical results," Composite Structures, Vol. 129, pp. 177–201.
7. Atteshamuddin S. Sayyad, Shinde B. M. and Ghugal Y. M., 2013, "Thermoelastic Bending Analysis of Orthotropic Plates using Hyperbolic Shear Deformation Theory," Composites Mechanics, Vol. 4(3), pp. 257–278.
8. Reddy J. N. and Hsu Y. N., 1980, "Effects of Shear Deformation and Anisotropy on the Thermal Bending of Layered Composite Plates," Journal of Thermal Stresses, Vol. 3, pp. 475-493.
9. Ghugal, Y. M., Kulkarni, S. K., 2011, Thermal stress analysis of cross-ply laminated plates using refinedshear deformation theory, Journal of Experimental and Applied Mechanics 2, pp. 47–66.
10. Sayyad, A. S., Ghugal, Y. M., 2012, Buckling analysis of thick isotropic plates by using exponential shear deformation theory, Applied and Computational Mechanics, Vol. 6(2), pp. 185–196.

