Kenneth K. Sirma et al. International Journal of Recent Research Aspects ISSN: 2349-7688, Vol. 8, Issue 4, Dec 2021, pp. 17-22

Quasi-Particle Energy of Atomic Nuclei with Neutron Excess (N>Z)

Kenneth K. Sirma[1], Hezekiah K. Cherop[2] and Paul Seurey[2]

[1]Department of Renewable energy and Technology, Turkana University College, P.O Box 69-30500, Lodwar, Kenya.

[2]Department of Physics, University of Eldoret, P.O Box 1125-30100, Eldoret, Kenya.

For correspondence: khannak700@gmail.com

Abstract- **Assuming that the core of a large finite nucleus in which the neutron number (N) is quite large compared to the proton number (Z), is composed of neutron-proton pairs (which are deutrons and bosons) and the unpaired neutrons stay in the surface region (neutron skin region), A model is proposed in which the deutrons in the core interact with the neutrons in the surface region. The nucleus is considered as an assembly of a mixture of interacting bosons (deutrons) and fermions (neutrons) and the model Hamiltonian is diagonalized to calculate the quasi-particle energy of the interacting system. The low value of the quasi-particle energy per neutron when compared to the large binding fraction can lead to a march towards the dripline for the neutrons.**

Keywords: Deutron, Fermions, Quasi-particle energy, Neutron skin.

I. Introduction

Superconductivity was first discovered by Kamerlingh onnes in 1911 [1]. The microscopic explanation of superconductivity was based on Cooper pair formation [2], and a microscopic theory was proposed [3]. A number of applications of this theory to nuclear structure were proposed [4,5] in which the nucleons are supposed to form pairs such that the pairs condense to a zero-momentum state (k=0). A number of experimental and theoretical studies were done in which the role of nucleon pairing was studied in finite nuclei and infinitely extended nuclear systems, such as , neutron star matter, infinite nuclear matter (symmetric and asymmetric). The role of pairing in studying superfluidity of such systems is spread over many years [6, 7, 8]. There is sufficient evidence to show that the pairing between particles in physical systems (nuclear physics and condensed matter physics) and quantum-many-body problem is an essential concept in calculating properties of many-body systems. Experimental data received in the field of astrophysics via a series of X-ray satellites; such as thermal, gravitational and optical (data) emissions from neutron stars has led to more active studies on the properties of finite nuclei with N>Z and in super-heavy-nuclei (SHN) with N>>Z, and also in designer nuclei with N>>Z (such as ${}^{11}_{3}Li$) [9,10,11]. In a number of laboratories with newly designed capabilities of radio-active beam and heavy-ion facilities, tremendous progress has been made in the exploration and fabrication of nuclei far from stability, with a particular focus on neutron-rich nuclei. [12, 13, 14, 15]. Pairing between nucleons plays an important role in modelling the structure and behaviour of these newly discovered nuclei.

Infact, there have been two theories that have been developed to obtain approximate solution to the quantum-many-body problem. One is the BCS theory (Bardeen-cooper-Schrieffer theory) [3] that has been used to study nuclear structure [4-8]. The second method is the Hartree-Fock (HF) theory [16] that has been used to describe various nuclear ground –state properties, but this theory does not include pairing interaction explicity. A proper method to include the pairing interaction into the many-body problem will be to solve the Hartree-Fock-Bogolubov (HFB) equations [17]. Another method to study the properties of the finite nuclei with large N (N>>Z) is to assume that the nucleus is composed of a core containing Z neutron-proton pairs (which are essentially deutrons and are bosons) surround by a neutron skin containing (N-Z) neutrons [11]. Infact, since deutrons in the core are bosons, and the neutrons in the neutron skin region are fermions, the nucleus as an assembly can be considered as an interacting system of bosons and fermions, and the quasi-particle energy of such an assembly can be calculated [18,19].

Looking at the periodic table, it shows that for light nuclei (low mass number A), neutron number N is equal to proton number Z(N=Z). In these nuclei, protons and neutrons occupy the same shell model orbitals. Thus there is a large spatial overlap between neutrons and protons single-particle wave-functions and this can result in large neutron-proton correlations (np-correlations) leading to substantial np-pairing. But as A increases, number of neutrons (N) increases faster than the number of protons (Z) such that N>Z, and it is found that we get better knowledge about nuclear pairing from nuclei in which N>Z in which the isotopic spin, T=1, neutron-proton (np) and proton-proton (pp) pairing dominates [20].

There seems to be a link between superfluidity in infinite neutron star matter and the spectra of finite nuclei [20]. It is the ${}^{1}s_0$ partial wave of the nucleon-nucleon interaction that provides the link. In another calculation [21], using the appropriate strength of the tensor force for deutron and the exact ${}^{1}s_0$ nucleon nucleon (n-n) scattering length, correct binding energy of the triton (E_B =-8.514 MeV) and the charge root-mean-square (rms) radius (r=1.72fm) were calculated

[18,21]. Even the correct value for charge r.m.s radius for ${}^{3}_{2}He$ (r=1.90fm) was calculated [21].
Heavy nuclei are supposed to develop a neutron-rich skin around the core of the nucleus (like in neutron stars), and in this skin region many of the neutrons collect near the surface of the nucleus. This skin thickness is strongly affected by the density dependence of the symmetry energy near saturation density. An accurate measurement of the thickness of the neutron –skin will provide information as to how the density dependence depends on the nuclear symmetry energy. Such measurements of the neutron-skin thickness should be independent of the nuclear model used [22].
It is to be noted that for spherical nuclei, such as ${}^{40}_{20}Ca$ (N=Z), surface tension favours the formation of a spherical nucleus of uniform equilibrium density and there is hardly any neutron skin. Whereas for nuclei with N>Z (heavier and larger nuclei) Coulomb repulsion is more than compensated by the symmetry energy, and this results in the emergence of neutron-skin. Thus in ${}^{208}_{82}Pb$, extra 44 neutrons should stay in the neutron-skin or in the core is a matter that is not settled yet [23].
However, in this manuscript, calculations have been done assuming that for nuclei in which N>Z, the extra neutrons (N-Z) constitute the neutron-skin, and the core of the nucleus is composed of neutron-proton pairs whose number equals to Z. It can, therefore be concluded that the neutron-proton pair interaction substantially contributes to the binding energy of atomic nuclei whether N=Z or N>Z. For designer nuclei or the nuclei in which either N>Z or Z>N, when the core is composed of neutron-proton pairs and the surface region is composed of unpaired protons (Z>N) or neutrons (N>Z), the core can be assumed to be composed of deutrons which are bosons (the spin of the deutron is integral and thus it is a boson), and the surface thickness or surface region of the nucleus will be composed of neutrons or protons which are fermions. The nucleus as a whole can now be treated as an assembly of interacting bosons and fermions

II. THEORY

In the proposed structure of a nucleus, we have an assembly of deutrons (bosons) in the core of the nucleus, and the surface region of the nucleus or the neutron skin of the nucleus (N>Z) is composed of fermions. Thus the nucleus as a whole is treated as an assembly of a mixture of interacting bosons and fermions. The model Hamiltonian, H, for such an assembly can be written as,

$$H=H_B+H_F+H_{BF} \qquad (1)$$

Where
H_B=Hamiltonian for bosons
H_F=Hamiltonian for fermions
H_{BF}= Hamiltonian for interacting bosons and fermions.
In terms of creation operators (a^+_{kB}, a^+_{kF}) and annihilation operators ($a_{kB,}$ a_{kF}) for bosons and fermions, the expression for H can be written as,
$H=H_B+H_F+H_{BF}$

$$H=\sum_K \in_{KB}\, a^+_{kB}a_{kB}+\sum_K \in_{KF}\, a^+_{KF}a_{KF} + \frac{1}{2}\sum G_{BF}\, a^+_{k1B}a^+_{k2F}a_{k_2'F}a_{k_1'B} \qquad (2)$$

In the last term in Eqn (2), momentum conservation is assumed such that the summation is carried over all values of k_1, k_2, k_1', k_2' so that,

$$k_1+ k_2 = k_1'+ k_2' \qquad (3)$$

In Eqn (2) it is emphasized that a boson with momentum k_1' is destroyed and a boson with momentum k_1 is created leading to a momentum transfer of k_1-k_1'. Similarly a fermion with momentum k_2' is destroyed and a fermion with momentum k_2 is created leading to a momentum transfer of (k_2-k_2'). For conservation of momentum, these two values of momentum transfer must be equal in magnitude but opposite in sign, i.e,
k_1-k_1'=-(k_2-k_2')=- k_2+k_2'
or

$$k_1+ k_2 = k_1'+ k_2' \qquad (4)$$

Eqn (4) shows that the sum of the momentum of a pair of boson and fermion before and after collision (interaction) must be the same. Due to the interaction between the boson and fermion, there will be different combinations of the k values.
The Hamiltonian H in Eqn (2) is diagonalized by using the canonical transformation developed for a mixture of bosons and fermions [19, 22], such that,

$$a^+_{kF}a_{kB}=\frac{1}{(u_k^2-v_k^2)}(v_k\mathcal{L}_k^+ + u_k\mathcal{L}_k) \qquad (5)$$

$$a^+_{kB}a_{kF}=\frac{1}{(u_k^2-v_k^2)}(u_k\mathcal{L}_k^+ + v_k\mathcal{L}_k) \qquad (6)$$

After going through lengthy calculations [19], the expression for the quasi-particle energy, E, of an assembly of a mixture of interacting bosons and fermions is,

$$E = \frac{1}{2}\left[\frac{4\pi a_{BF}\hbar^2}{m_{BF}}\right] n_B^2 n_F^2 \qquad (7)$$

Where,
a_{BF}= scattering length between interacting bosons and fermions
m_{BF} = reduced mass for a pair of boson and fermion
$\hbar = \frac{h}{2\pi}$, h= planck's constant.
n_B= particle number density of bosons
n_F= particle number density of fermions
Eq (7) is used for the calculations of E which is the binding energy for neutrons in the neutron skin region of a nucleus with N>Z.

III. RESULTS AND DISCUSSION

Now it is assumed that at any time the number of particles taking part in the scattering of deutron-neutron interacting system can not be more than the nuclear matter particle number density (which is the saturation density) which is, N, i.e

$$N=1.95X10^{38} \text{ particles cm}^{-3} \qquad (8)$$

m_n= 1.008 amu = Mass of the neutron.
m_d = 2.014 amu = Mass of the deutron.
$m_{BF} = \frac{m_n m_d}{m_n+m_d}$= 0.67X10^{-24}gm
a_{BF}= 0.65fm, 0.67fm, 0.72fm, 0.94fm, and 6.3fm.
These are S-wave scattering lengths [18] for interacting deutrons and neutrons
Now considering a nucleus ${}^{235}_{92}U$, the value of,

$$E=74.3\text{MeV} \qquad (9)$$

The energy per neutron in the surface region or neutron skin will be, $\in_n$

$$\epsilon_n = \frac{E}{51} = \frac{74.3}{51}\ \text{MeV} \qquad (10)$$

The quasi particle energy and the energy per neutron have been calculated for the following nuclei.

NUCLEI	A	Z	N	a_{BF}	ϵ (MeV)
U	234	92	142	5	1.17935
U	235	92	143	5	1.156225
U	236	92	144	5	1.13399
U	238	92	146	5	1.091991
U	237	92	145	5	1.112594
U	239	92	147	5	1.072136
U	240	92	148	5	1.052991
U	241	92	149	5	1.034518
U	242	92	150	5	1.016681
U	243	92	151	5	0.999449
Np	229	93	136	5.2	1.426191
Np	231	93	138	5.2	1.362804
Np	233	93	140	5.2	1.304813
Np	234	93	141	5.2	1.277629
Np	235	93	142	5.2	1.251555
Np	236	93	143	5.2	1.226524
Np	237	93	144	5.2	1.202475
Np	238	93	145	5.2	1.17935
Np	239	93	146	5.2	1.157098
Np	240	93	147	5.2	1.13567
Pu	232	94	138	5.4	1.447384
Pu	233	94	139	5.4	1.41522
Pu	234	94	140	5.4	1.384454
Pu	235	94	141	5.4	1.354998
Pu	236	94	142	5.4	1.326769
Pu	237	94	143	5.4	1.299692
Pu	238	94	144	5.4	1.273698
Pu	239	94	145	5.4	1.248724
Pu	240	94	146	5.4	1.22471
Pu	241	94	147	5.4	1.201602
Pu	242	94	148	5.4	1.17935
Pu	244	94	150	5.4	1.13723
Am	237	95	142	5.5	1.38009
Am	238	95	143	5.5	1.351339
Am	239	95	144	5.5	1.32376
Am	240	95	145	5.5	1.297285
Am	241	95	146	5.5	1.271848
Am	242	95	147	5.5	1.247389
Am	243	95	148	5.5	1.223854
Cm	238	96	142	5.6	1.43573
Cm	239	96	143	5.6	1.405183
Cm	240	96	144	5.6	1.375908
Cm	241	96	145	5.6	1.347829
Cm	242	96	146	5.6	1.320872
Cm	243	96	147	5.6	1.294973
Cm	244	96	148	5.6	1.270069
Cm	245	96	149	5.6	1.246106
Cm	246	96	150	5.6	1.22303
Cm	247	96	151	5.6	1.200793
Cm	248	96	152	5.6	1.17935
Cm	250	96	154	5.6	1.138683
Bk	238	97	141	5.6	1.500991
Bk	239	97	142	5.6	1.467636
Bk	245	97	148	5.6	1.294973
Bk	246	97	149	5.6	1.270069
Bk	247	97	150	5.6	1.246106
Bk	248	97	151	5.6	1.22303
Bk	249	97	152	5.6	1.200793
Bk	250	97	153	5.6	1.17935
Cf	240	98	142	5.6	1.500991
Cf	241	98	143	5.6	1.467636
Cf	242	98	144	5.6	1.43573
Cf	243	98	145	5.6	1.405183
Cf	244	98	146	5.6	1.375908
Cf	245	98	147	5.6	1.347829
Cf	246	98	148	5.6	1.320872
Cf	248	98	150	5.6	1.270069
Cf	249	98	151	5.6	1.246106
Cf	250	98	152	5.6	1.22303
Cf	251	98	153	5.6	1.200793
Cf	252	98	154	5.6	1.17935
Cf	253	98	155	5.6	1.15866
Cf	254	98	156	5.6	1.138683
Es	240	99	141	5.7	1.600546
Es	241	99	142	5.7	1.563324
Es	242	99	143	5.7	1.527794
Es	243	99	144	5.7	1.493843
Es	245	99	146	5.7	1.430276
Es	246	99	147	5.7	1.400478
Es	247	99	148	5.7	1.371897
Es	248	99	149	5.7	1.344459
Es	249	99	150	5.7	1.318097

Es	251	99	152	5.7	1.268358
Es	252	99	153	5.7	1.244869
Es	253	99	154	5.7	1.222235
Es	254	99	155	5.7	1.20041
Es	255	99	156	5.7	1.17935
Fm	241	100	141	5.7	1.639584
Fm	242	100	142	5.7	1.600546
Fm	243	100	143	5.7	1.563324
Fm	244	100	144	5.7	1.527794
Fm	245	100	145	5.7	1.493843
Fm	246	100	146	5.7	1.461368
Fm	247	100	147	5.7	1.430276
Fm	248	100	148	5.7	1.400478
Fm	249	100	149	5.7	1.371897
Fm	250	100	150	5.7	1.344459
Fm	251	100	151	5.7	1.318097
Fm	252	100	152	5.7	1.292749
Fm	253	100	153	5.7	1.268358
Fm	254	100	154	5.7	1.244869
Fm	255	100	155	5.7	1.222235
Fm	256	100	156	5.7	1.20041
Fm	257	100	157	5.7	1.17935
Fm	258	100	158	5.7	1.159016
Fm	259	100	159	5.7	1.139372
Fm	260	100	160	5.7	1.120383
Md	248	101	147	5.8	1.487007
Md	252	101	151	5.8	1.368046
Md	257	101	156	5.8	1.243678
Md	258	101	157	5.8	1.22147
Md	260	101	159	5.8	1.17935
No	250	102	148	5.8	1.487007
No	251	102	149	5.8	1.455368
No	252	102	150	5.8	1.425048
No	253	102	151	5.8	1.395965
No	254	102	152	5.8	1.368046
No	255	102	153	5.8	1.341222
No	256	102	154	5.8	1.315429
No	257	102	155	5.8	1.290609
No	258	102	156	5.8	1.266709
No	259	102	157	5.8	1.243678
No	260	102	158	5.8	1.22147
No	262	102	160	5.8	1.17935
Lr	252	103	149	5.9	1.512645

Lr	253	103	150	5.9	1.480461
Lr	254	103	151	5.9	1.449618
Lr	255	103	152	5.9	1.420034
Lr	256	103	153	5.9	1.391633
Lr	257	103	154	5.9	1.364346
Lr	258	103	155	5.9	1.338109
Lr	259	103	156	5.9	1.312861
Lr	260	103	157	5.9	1.288549
Lr	261	103	158	5.9	1.265121
Lr	262	103	159	5.9	1.242529
Rf	253	104	149	5.9	1.546259
Rf	254	104	150	5.9	1.512645
Rf	255	104	151	5.9	1.480461
Rf	256	104	152	5.9	1.449618
Rf	257	104	153	5.9	1.420034
Rf	258	104	154	5.9	1.391633
Rf	259	104	155	5.9	1.364346
Rf	260	104	156	5.9	1.338109
Rf	261	104	157	5.9	1.312861
Rf	262	104	158	5.9	1.288549
Rf	263	104	159	5.9	1.265121
Rf	267	104	163	5.9	1.17935
Db	256	105	151	6	1.538283
Db	257	105	152	6	1.505553
Db	258	105	153	6	1.474188
Db	259	105	154	6	1.444102
Db	260	105	155	6	1.41522
Db	261	105	156	6	1.387471
Db	262	105	157	6	1.360788
Db	263	105	158	6	1.335113
Db	266	105	161	6	1.263589
Db	267	105	162	6	1.241421
Db	268	105	163	6	1.220017
Db	269	105	164	6	1.199339
Db	270	105	165	6	1.17935
Sg	258	106	152	6.1	1.563921
Sg	259	106	153	6.1	1.530646
Sg	260	106	154	6.1	1.498757
Sg	261	106	155	6.1	1.46817
Sg	262	106	156	6.1	1.438807
Sg	263	106	157	6.1	1.410595
Sg	264	106	158	6.1	1.383468
Sg	265	106	159	6.1	1.357365

Sg	266	106	160	6.1	1.332229
Sg	267	106	161	6.1	1.308006
Sg	271	106	165	6.1	1.219328
Sg	272	106	166	6.1	1.199006
Sg	273	106	167	6.1	1.17935
Bh	260	107	153	6.2	1.589559
Bh	261	107	154	6.2	1.555738
Bh	262	107	155	6.2	1.523327
Bh	263	107	156	6.2	1.492239
Bh	264	107	157	6.2	1.462394
Bh	265	107	158	6.2	1.43372
Bh	266	107	159	6.2	1.406148
Bh	267	107	160	6.2	1.379617
Bh	270	107	163	6.2	1.305709
Bh	272	107	165	6.2	1.260684
Bh	273	107	166	6.2	1.239317
Bh	274	107	167	6.2	1.218662
Bh	275	107	168	6.2	1.198684
Hs	263	108	155	6.3	1.580831
Hs	264	108	156	6.3	1.547897
Hs	265	108	157	6.3	1.516307
Hs	266	108	158	6.3	1.485981
Hs	267	108	159	6.3	1.456844
Hs	269	108	161	6.3	1.401869
Hs	270	108	162	6.3	1.375908
Hs	271	108	163	6.3	1.350892
Hs	275	108	167	6.3	1.259306
Hs	277	108	169	6.3	1.218017
Mt	266	109	157	6.3	1.547897
Mt	268	109	159	6.3	1.485981
Mt	270	109	161	6.3	1.428828
Mt	274	109	165	6.3	1.326769
Mt	275	109	166	6.3	1.303492
Mt	276	109	167	6.3	1.281018
Mt	277	109	168	6.3	1.259306
Mt	278	109	169	6.3	1.238318
Ds	267	110	157	6.3	1.580831
Ds	269	110	159	6.3	1.516307
Ds	270	110	160	6.3	1.485981
Ds	271	110	161	6.3	1.456844
Ds	272	110	162	6.3	1.428828
Ds	273	110	163	6.3	1.401869
Ds	274	110	164	6.3	1.375908

Ds	279	110	169	6.3	1.259306
Ds	281	110	171	6.3	1.218017
Rg	272	111	161	6.3	1.485981
Rg	274	111	163	6.3	1.428828
Rg	278	111	167	6.3	1.326769
Rg	279	111	168	6.3	1.303492
Rg	280	111	169	6.3	1.281018
Rg	281	111	170	6.3	1.259306
Rg	282	111	171	6.3	1.238318
Cn	277	112	165	6.3	1.401869
Cn	280	112	168	6.3	1.326769
Cn	281	112	169	6.3	1.303492
Cn	282	112	170	6.3	1.281018
Cn	283	112	171	6.3	1.259306
Cn	284	112	172	6.3	1.238318
Cn	285	112	173	6.3	1.218017
Ed	278	113	165	6.3	1.428828
Ed	282	113	169	6.3	1.326769
Ed	283	113	170	6.3	1.303492
Ed	284	113	171	6.3	1.281018
Ed	285	113	172	6.3	1.259306
Ed	286	113	173	6.3	1.238318
Ed	287	113	174	6.3	1.218017
Fl	286	114	172	6.3	1.281018
Fl	287	114	173	6.3	1.259306
Fl	288	114	174	6.3	1.238318
Fl	289	114	175	6.3	1.218017
Ef	287	115	172	6.3	1.303492
Ef	288	115	173	6.3	1.281018
Ef	289	115	174	6.3	1.259306
Ef	290	115	175	6.3	1.238318
Ef	291	115	176	6.3	1.218017
Lv	289	116	173	6.3	1.303492
Lv	290	116	174	6.3	1.281018
Lv	291	116	175	6.3	1.259306
Lv	292	116	176	6.3	1.238318
Lv	293	116	177	6.3	1.218017
Eh	292	117	175	6.3	1.281018
Eh	293	117	176	6.3	1.259306
Eh	294	117	177	6.3	1.238318
Ei	293	118	175	6.3	1.303492
Ei	294	118	176	6.3	1.281018
Ei	295	118	177	6.3	1.259306

IV. CONCLUSIONS

It is clear that the value of $\in_n$ depends on the magnitude of a_{BF} and the number of neutrons in the neutron skin region. The magnitude of $\in_n$ is small compared to the binding fraction, f, which is the order of 7.8MeV. It should be small compared to f since the excess neutrons in the surface region must be loosely bound, and as the number of neutrons is increased in the neutron skin region, it should lead to neutron dripline at which the separation energy of the neutron becomes zero, or existence of the nucleus ends. For comparison, calculations can be done for the super heavy nuclei (SHN), such as, ${}^{262}_{105}Db$ (Dubrium) and ${}^{272}_{111}Uuu$ (Ununnium).

V. ACKNOWLEDGEMENT

Thanks are due to Professor K.M. KHANNA, department of physics, university of Eldoret, Eldoret, Kenya for helpful discussions and assistance during the preparations of the manuscript.

REFERENCES

[1]. Kamerlingh, H. Onnes. Leiden. Commun. 122b, 124C (1911)

[2]. Cooper, L.N. Bound Electrons pairs in a Degenerate Fermi Gas. Phys. Rev. 104 (1956) 1189.

[3]. Bardeen, J., Cooper, L. N., and Schrieffer. Phys. Rev. 1175 (1957) 108

[4]. Belyaev, S.T. Mat.Fys.Medd.Dan. Vid. Selsk. 641 (1959) 31

[5]. Migdal, A., Nucl. Phys. 655(1959) 13 and Sov. Phys. JETP.176 (1960)12

[6]. Cooper, L.N., Mills, R.L., and Sessler, A.M. Phys. Rev 1377(1959)114

[7]. Emery, V.J., and Sessler, A.M. Phys. Rev. 248(1960) 119

[8]. Khanna, K.M. Prog. Theor. Phys. Japan. Vol.28, (1) 205 (1962) and Proc. Nat. Inst. Of Sc. India. Vol. 29 A (1), 205(1963).

[9]. Hezekiah, C., Muguro, K., and Khanna, K. The Role of Shell Model in Determining Pairing Interaction in Nuclei. IJRAR Vol.6, (4), (2019)10-15.

[10]. Moller, P., and Nix, J.R. Nuclear Pairing Models. Nuclear Physics A536 (1992)20-60.

[11]. K. K. Sirma, L. S. Chelimo, K. M. Khanna. Interaction between neutron-proton core and neutron skin region in super heavy nuclei. WSN 144 (2020) 243-265.

[12]. Mueller, A.C., and Sherril, B.M, Ann. Rev. Nucl. Parts. Phys. 43(1993) 529.

[13]. Risagar, K., Rev. Mod. Phys. 66(1994)1105.

[14]. Oganessian, Yu. Ts., and Utyonkov, V.K. Superheavy Nuclei from ${}^{48}_{20}Ca$ induced reactions. Nuclear Physics A944 (2015) 62-98.

[15]. Michimasa, S. *et.al.* Magic Nature of neutrons in ${}^{54}_{20}Ca$ first mass measurements of ${}^{55-57}_{20}Ca$. Phys. Rev. Letts. 121, 022506 (2018).

[16]. Bogoliubov, P., and Flocard, H. Ann. Rev. Nucl. Sc. 28(1978) 523.

[17]. Bogoliubov, N.N. Dolk. Arad. Nank. SSR. 119 (1959) 244.

[18]. Fukukawa, K., and Y-Fujiwara. Effective-range Expansion of Neutron-Deutron scattering studied by a Quark-Model Nonlocal Gaussian Potential. Progress of Theoretical Physics, Vol. 125 (5), (2011).

[19]. Obota, S.E., and Khanna, K.M. Quasi-particle Energy of a Mixture of Bosons and Fermions. SITA Vol. 22, No.3, 4 (2020)

[20]. Dean, D.J., and Hjorth-Jensen, M. Rev. Mod. Phys. Vol. 75(2003) 607.

[21]. Y-Fujiwara., Miyagawa, K., Nemura, H. Phys.Rev.C66 (2002) 014002.

[22]. Khanna, K.M., Stanslous, E.O., Tonui, K.J. Canonical transformation for Amixture of Bosons and Fermions. Scientific Israel Technological Advantage Vol. 21, No. 5, 6, (2019) 90-97.

